Calculation of the position determination error

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Abstract

The fiber-to-target convergence performance of PFI depends on the ratio of *allowable fiber placement error* to *position determination error* allocations. *Allowable fiber placement error* is the distance between the measured position of the fiber and the target location, while *position determination error* quantifies how well we know the position of the fiber in PFI coordinates. For the fibers to converge on the target location quickly, the position determination error needs to be small compared to the allotted fiber placement accuracy. When the position determination error is not smaller than the allowable fiber placement error, convergence to the target becomes a stochastic, rather than controlled, process.

While the allowable fiber placement error is entirely an error budget allocation, the position determination error is a function of PFI to metrology camera path-induced errors, metrology camera measurement error, and PFI calibration (V&V) measurement errors. This document describes how the position determination error is calculated from these fundamental error terms.

1 Introduction: PFI (X, Y) error budget

The purpose of PFI is to position optical fibers at the focal-plane locations of astronomical objects for a given telescope pointing. The alignment of fibers to object images is limited by two separable terms: the *target location error*, which is the error in the position of the astronomical object image in PFI focal plane coordinates; and the *fiber location error*, which is the error in the position of the optical fibers in PFI focal plane coordinates. This document is primarily concerned with the parts of the fiber location error that affect fiber configuration. Aside from acknowledging its existence and recognizing that it takes part of the PFI (X, Y) error budget, I will say no more about the target location error.

Fiber location error consists of (1) position determination error, (2) allowable fiber placement error, and (3) drift errors due to mechanical instabilities. During PFI configuration, which is allocated 105 seconds, we are only concerned with the position determination error and allowable fiber placement error. Drift errors are assumed to be small over the configuration time, but need to be accounted for over the time of an observation (∼15–20 minutes). Figure 1 illustrates the distinction between position determination error and allowable fiber placement error: position determination error quantifies how close the measured fiber position is to the true fiber position, while the allowable fiber location error defines the maximum acceptable distance between the measured location of the fiber and the target location.

The error budget for optical fiber to astronomical alignment is tracked in the Excel file "Alignment Error BudgetYYYY-MM-DD.xls" by David Braun (JPL). The partitioning of the alignment error described here differs from that of Braun's error budget. I have partitioned the error budget based on operational considerations (fiber location vs. target location and configuration placement errors vs. instability induced errors), while Braun partitions the error budget by the deliverable hardware. Position determination error and allowable fiber placement error are each a branche of "PFI Design and Installation Errors + Uncompensated Instabilities" in the PFS (X, Y) error budget.

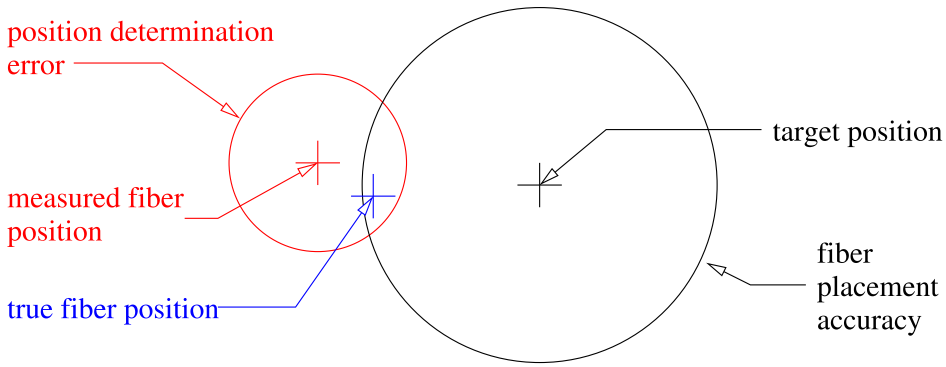

Figure 1: Position determination error and allowable fiber placement error shown relative to the astronomical target position in PFI focal coordinates. Note that the true fiber position is unknown. Also note that the position determination error is specified by its Gaussian (1σ) standard deviation, while the fiber placement accuracy is a maximum allowable measured distance to the target location. In the case shown here, the fiber placement is not on-target.

This document describes how we calculate the position determination error. In Section 2, I will first enumerate the error terms that contribute to position determination error. In Section 3, I will discuss methods for determining the positions of fibers in PFI coordinates from the Metrology Camera System (MCS) image. Finally, in Section 4, I will show how to calculate position determination error from the contributing terms when biharmonic spline interpolation is used for the MCS to PFI coordinate transformation.

2 Contributions to position determination error

We determine the position of a fiber in PFI focal plane coordinates by transforming metrology camera (MCS) pixel coordinates into PFI focal plane coordinates using a model of the back-illuminated WFC distortion function ($\mathrm{WFC}_{\mathrm{MCS}}$ distortion). It is convenient, for later discussions, to divide the error terms into those that contribute to the MCS measurement error and those that contribute to error in reference fiber[1] position knowledge. In the first category, we have *WFC_{MCS} distortion error*, *dome seeing* and *centroiding errors*. In the second category, we have *calibration error* of the reference fiber positions and, for science fibers specifically, their *home-position repeatability*. With the exception of WFC distortion error, these error terms are largely

[1] I am introducing this term to describe fiducial fibers and homed science fibers

self-explanatory.

Dome seeing is the effect of turbulence between PFI and the MCS — its effect was measured under various conditions by Tamura and Wang in December, 2012.

Centroiding error is a function of the size of a fiber image on the MCS and the signal to noise ratio in the image.

Calibration error is the error in locating the positions of the fiducial fibers and the science fiber home positions during integration and any time after the PFI bench is serviced.

Home position repeatability only affects science fibers when we use them in their home positions as reference fibers.

$\mathbf{WFC_{MCS}}$ distortion error is the displacement between the distortion model, our best estimate of the back-illuminated WFC distortion, and the true distortion. The distortion model, and therefore the distortion error as well, is dependent on the position determination method. Position determination methods are discussed in Section 3, while the dependence of distortion error on the position determination method is detailed in Appendix A.

Physically, the main source of $\mathrm{WFC_{MCS}}$ distortion error is short length-scale (high frequency) optical figure errors due to polishing. Note that the front-illuminated WFC distortion that Hyper SuPrime Cam (HSC) sees is largely immune to high frequency figure error because the footprint of astronomical targets on WFC covers all (or most) of the WFC optics. The net effect of high frequency figure error on HSC is some blur on the image of each object. In contrast, the back-illuminated WFC distortion that MCS sees is expected to be significantly affected by high frequency figure error. The footprint of each backlit fiber on WFC has a length scale no larger than a few millimeters, so the location of the fiber image on MCS is heavily influenced by the local WFC figure error. Each fiber image would then be translated in some unpredictable direction at MCS depending on the local WFC figure error that it encounters.

3 Position determination method

In order to determine the PFI focal plane coordinates of science fibers from the MCS centroid coordinates of the back-lit fiber image, we need a measurement of the back-illuminated WFC distortion seen by MCS, which I will call the distortion map. The distortion map is a measurement of the MCS coordinates of known PFI focal plane coordinates — most likely, back-it reference fibers. The distortion map may be sparsely or densely sampled, depending on wheter or not the homed science fibers are used to determine the map. Differences in position determination methods amount to differences in how the distortion model is determined from the distortion map.

I have investigated position determination using both a closed-form least squares fit and iterpolation. In the simulation of the MCS images, PFI fiber positions are first defined, and then the PFI bench is rotated with respect to the MCS coordinate system. The MCS image is constructed by radially distorting the PFI fiber positions and then individually translating each fiber position, with the translation vectors drawn from a 2-D Gaussian distribution. The rotation and distortion centers are not coincident with each other and are unknowns to be determined by the position determination method, if need be. The distribution width of the random translations is the quadrature sum of all of the relevant terms described in Section 2.

3.1 Closed-form distortion model with least squares fit

The project baseline method for determining fiber positions is to parameterize the distortion function with a closed-form model and determine the model parameters by a least squares fit of the model to the distortion map, using only the 153 fiducial fibers. The true distortion function is presumed to be a simple radial function, but as the function becomes more complicated, the model must also increase its complexity, making the least squares fit more difficult. The simplest distortion model assumes that non-radial (e.g., distortions due to the atmospheric dispersion corrector) and random (e.g., WFC figure-error induced) errors do not contribute significantly to the distortion function ($\ll 1$ μm). Presently, the magnitudes of these errors are unknown, but they pose a serious risk to the viability of modelling the distortion function with a closed-form expression, and hence pose a serious risk to our ability to determine PFI fiber positions.

Even if we assume that the distortion is purely radial, we do not have *a priori* knoweldge of the distortion center. Although it would be most natural to fit the distortion model to the distortion map using cylindrical coordinates, I can't do it without knowing the distortion center. Furthermore, error in the determination of the distortion center translates into position determination errors. Using the relative distortion function (Equation 4.3) from the 2009 Hyper SuPrime Cam Design Review, a 13 μm error in the distortion center translates into a 1 μm error in the fiber position determination at the edge of PFI (See Appendix B for details).

To test how well the distortion center can be recovered in a closed-form least squares fit, I distorted the PFI fiducial fibers with a third-order distortion function:

$$\delta_r = r_{\mathrm{MCS}} - r_{\mathrm{PFI}} = -6.2 \times 10^{-7} r_{\mathrm{PFI}}^3. \tag{3.1}$$

This function produces 6 mm of distortion at the edge of PFI, just like the function that HSC measured, but drops the 5th order term for simplicity. When this distortion function is applied to the PFI fiducial fibers, we need two additional terms for the distortion center (r_{DC}) and the offset between the coordinate systems ($r_{\mathrm{MCS\text{-}PFI}}$). The MCS position (in PFI-scaled coordinates) is, therefore:

$$r_{\mathrm{MCS}} = A(r_{\mathrm{PFI}} - r_{\mathrm{DC}})^3 + r_{\mathrm{MCS\text{-}PFI}}. \tag{3.2}$$

The distortion map is generated by taking these MCS positions and adding random Gaussian noise to their positions. This model is fit to the distortion map by separating the problem into its x and y components. Each Cartesian component is separately fit using a nonlinear least squares (trust-region) algorithm. For each component, there are four unknowns: A, $r_{x,\mathrm{DC}}$, $r_{y,\mathrm{DC}}$, and $r_{x,\mathrm{MCS\text{-}PFI}}$ or $r_{y,\mathrm{MCS\text{-}PFI}}$ (for the x and y components, respectively).

In 1000 simulations of this distortion model (with 5 μm random error in the MCS positions), the error in the distortion center ranged up to 40 μm, which translates into a $\sim$2 μm error at the edge of the field of view for this model. This example demonstrates that for simple functions, we can solve the distortion with the fiducial fibers alone; however, because of potential complications from non-radial and/or high-frequency errors, least squares fitting has not been pursued as vigorously as interpolation.

3.2 Interpolation

If we use both the science fiber home positions and the fiducials as reference fibers when we acquire the distortion map, then we have sufficient sampling density to use interpolation methods to define the distortion model. The main advantage of interpolation over a closed-form fit is, up to the Nyquist frequency of the distortion map [nominally, $1/(2 \times \text{fiber pitch})$], we don't need to know the functional form of the true distortion. This solves the problem of non-radial distortion and makes the issue of finding the distortion center irrelevant. It does not, however, completely cover the issue of high-frequency errors. Specifically, the $\mathrm{WFC}_{\mathrm{MCS}}$ distortion error is the integrated power spectral density (PSD) of the distortion function from the Nyquist frequency to 1/(diameter of a fiber footprint on WFC).

If we know the PSD of the distortion function, then we can adjust the Nyquist frequency of the distortion map, thereby balancing $\mathrm{WFC_{MCS}}$ distortion error with the other error term(s). If $\mathrm{WFC_{MCS}}$ distortion error is dominant, we can add multiple distortion maps to increase the sampling density. This has the effect of reducing $\mathrm{WFC_{MCS}}$ distortion error by reducing the bandwidth over which we integrate the PSD of the distortion function. If dome seeing, centroiding, and positioner errors are the dominant error terms, then we can average the distortion over several reference fibers. Averaging fibers reduces these errors by $1/\sqrt{N \text{ reference fibers}}$, but increase the $\mathrm{WFC_{MCS}}$ distortion error.

Matlab offers canned routines for linear, nearest neighbor, cubic spline, and biharmonic interpolation [see Sandwell, GRL(1987)]. Of the four, biharmonic interpolation is the most consistent in returning reasonable results (ie, no unexpected NaN's) and is currently the baseline method for position determination in my PFI fiber-to-target convergence simulations.

4 Calculation of the position determination error

For each moved fiber we use three reference fiber positions from the distortion map and the fiber position in the new image to determine the PFI coordinate of the fiber.

When we use interpolation to determine the PFI positions of non-homed science fibers, we iterpolate the current fiber position between the three nearest distortion map fiber positions. The position determination error is, therefore, the quadrature sum of the position errors from three reference fiber positions from the distortion map image (DM) and one from the current fiber location image (CFL):

$$\sigma_{\mathrm{PFI\ pos.}} = \sqrt{3\sigma_{\mathrm{DM}}^2 + \sigma_{\mathrm{CFL}}^2}. \tag{4.1}$$

The errors from these two images are different because the distortion map image has knowledge of the PFI positions of the fibers from the calibration information, but the current fiber location image does not.

For the distortion map (Figure 2), the reference fiber error is the quadrature sum of dome seeing, centroiding, calibration, and (for homed science

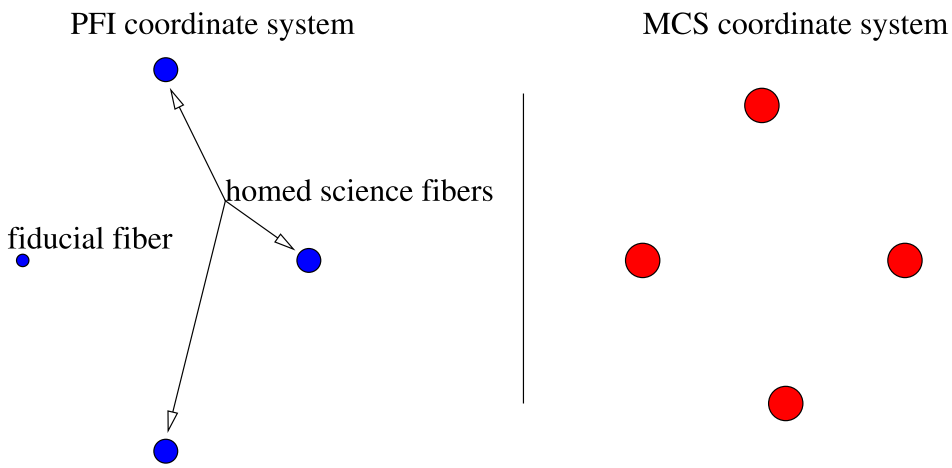

Figure 2: Distortion map. **Left:** The calibration data on the positions of homed science fibers and a single fiducial fiber with marker size indicating the measurement errors of the fiber positions. The science fibers have a larger error than the fiducial because of the additional contribution from home position repeatability. **Right:** MCS measurement of the same fibers. Error contributions in the MCS image include dome seeing and centroiding error. Because the distortion model is defined by this distortion map, $\mathrm{WFC_{MCS}}$ distortion error does not contribute to the measurement error.

fibers) the home-position repeatability:

$$\sigma^2_{\mathrm{DM}} = \begin{cases} \sigma^2_{\mathrm{dome}} + \sigma^2_{\mathrm{centroid}} + \sigma^2_{\mathrm{cal}}, & \text{for fiducial fibers} \\ \sigma^2_{\mathrm{dome}} + \sigma^2_{\mathrm{centroid}} + \sigma^2_{\mathrm{cal}} + \sigma^2_{\mathrm{home}}, & \text{for homed science fibers.} \end{cases} \tag{4.2}$$

The $\mathrm{WFC_{MCS}}$ distortion error does not enter into this term because, at the reference fiber locations, the distortion model is *defined* by the distortion map. $\mathrm{WFC_{MCS}}$ distortion error only exists when we are measuring science fibers at positions other than homed positions during reconfiguration.

This needs to be verified w/ numerical simulations.

Note that home position repeatability is not 2-D Gaussian distributed — it is distributed in a bow-tie pattern. Using $r_{\mathrm{arm}} = 2.375$ mm for the cobra arm length, and $\Delta\theta$ and $\Delta\phi$ for the lower and upper motor angular home position repeatabilities, the home position repeatability has a bow-tie distributed width $r_{\mathrm{arm}}\Delta\phi$ and height $r_{\mathrm{arm}}\Delta\phi\Delta\theta$. This means that propagating the home position repeatability in a quadrature sum is not strictly correct.

For the (off-home) current fiber location image (Figure 3), the image location error of a single fiber is the quadrature sum of dome seeing, centroiding

need to discuss semantics w/ DB

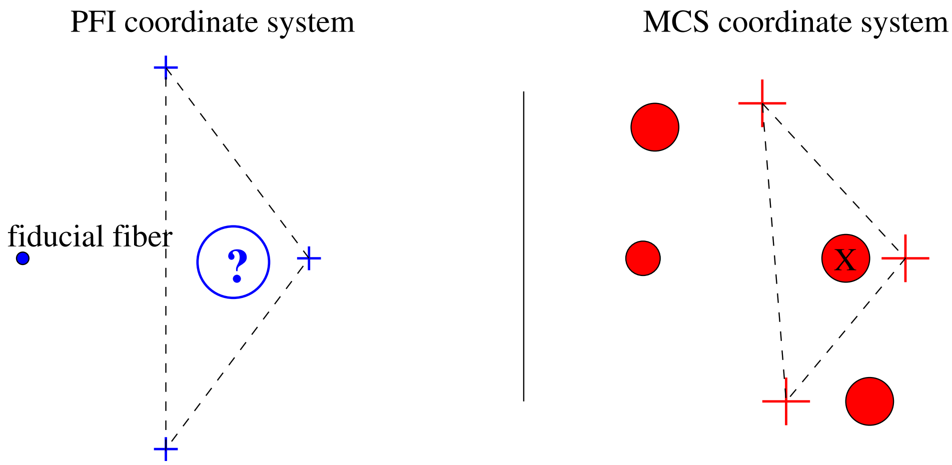

Figure 3: Post-move current fiber location (CFL) image. **Left:** After the science fibers are moved, without additional information, we only know the location of the fiducial fibers in PFI coordinates. The home positions of the science fibers are shown here as +'s. The "?" represents the to-be-calculated PFI position of one of the science fibers. **Right:** MCS measurement of the current fiber locations. Dome seeing and centroiding errors again contribute to the measurement error, but now we also have the $\mathrm{WFC_{MCS}}$ distortion error because the science fibers are no longer at known locations. In order to calculate the position of the fiber marked "X," we need to use its position in the feedback image and at least three distortion map fiber positions (marked with "+").

error, and $\mathrm{WFC_{MCS}}$ distortion error:

$$\sigma_{\mathrm{CFL}} = \sqrt{\sigma^2_{\mathrm{dome}} + \sigma^2_{\mathrm{centroid}} + \sigma^2_{\mathrm{WFC}}}. \tag{4.3}$$

Reference fiber position errors do not come into play here. On the other hand, because the distortion model (the interpolation) deviates from the true distortion when we are not on the distortion map's data, the $\mathrm{WFC_{MCS}}$ distortion error term does apply in this case.

A Distortion error power spectral density

This section is in-progress/incomplete.

Comparing the power spectral density of the distortion error for closed-form least squares fitting and interpolation highlights some important differences between the two methods.

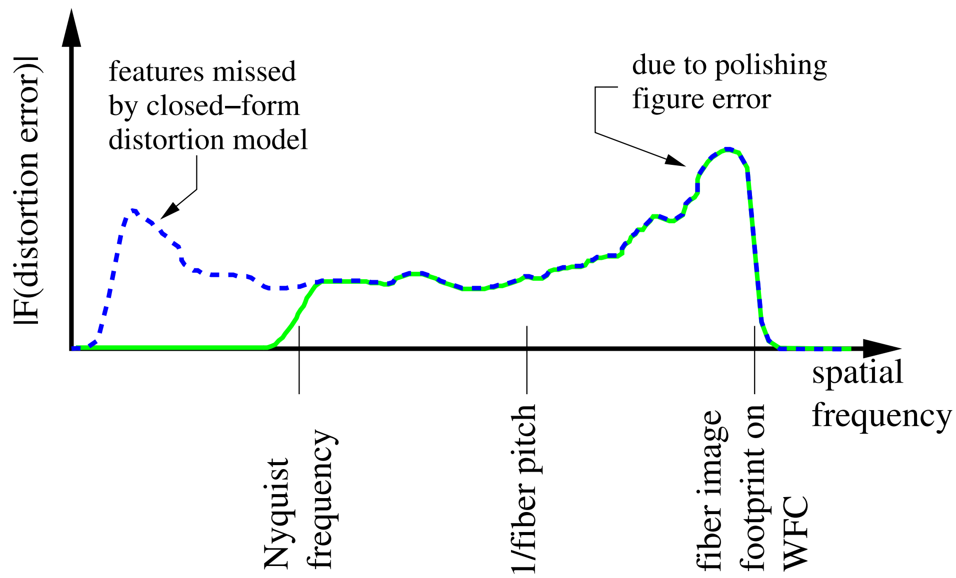

Figure 4: Distortion error power spectral density. Schematically shown here is the power spectral density (Fourier transform) of the distortion error. The error for interpolation is shown in green and the error for closed-form least squares fitting is shown as a dashed blue line. At the low frequency end, interpolation is only affected by the sampling frequency, while the closed-form solution picks up errors almost all the way to the length scale of the bench, depending on how well the model fits the true distortion.

- Interpolation DE shown in green, closed-form error in blue (dashed)
- distortion error is the integral of this function.
- Both PSDs have a high frequency cutoff at frequency 1/(footprint diameter)
- Interpolation has a low frequency cutoff at the Nyquist frequency. Worst case, this is $1/(2 \times \text{fiber pitch})$. This can be improved by taking multiple, offset images.
- The low frequency cutoff in the closed form case corresponds to the maximum frequency of the distortion model function.

B Distortion center knowledge requirement

TBD.

C Glossary

PSD: Power spectral density.

CFL: Current fiber location.

MCS: Metrology Camera System. Mounted at Cassegrain focus. This closes the Cobra position feedback loop.

WFC: Wide Field Corrector.

Distortion function: The true distortion of the location of a back-lit fiber in MCS coordinates.

Distortion map: A measurement of PFI reference fibers by MCS through WFC.

Distortion model: A model of the distortion function based on the distortion map. This is used to transform from MCS pixel coordinates to PFI focal plane coordinates.

Fiber location error: The distance between the true fiber location and the requested target location.

Allowable fiber placement error: The distance between the measured fiber location and the requested target location.

Postion determination error: The distance between the true fiber location and the measured fiber location.