BTO Control Math
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14 November, 2005 (rev 11)

1. Overview

This document describes the 200-inch Beam Transfer Optics (BTO) from a control perspective, and derives the necessary control relations for its tracking features. Figure 1 (Antonin's) shows the BTO schematically and labels the relevant pieces.

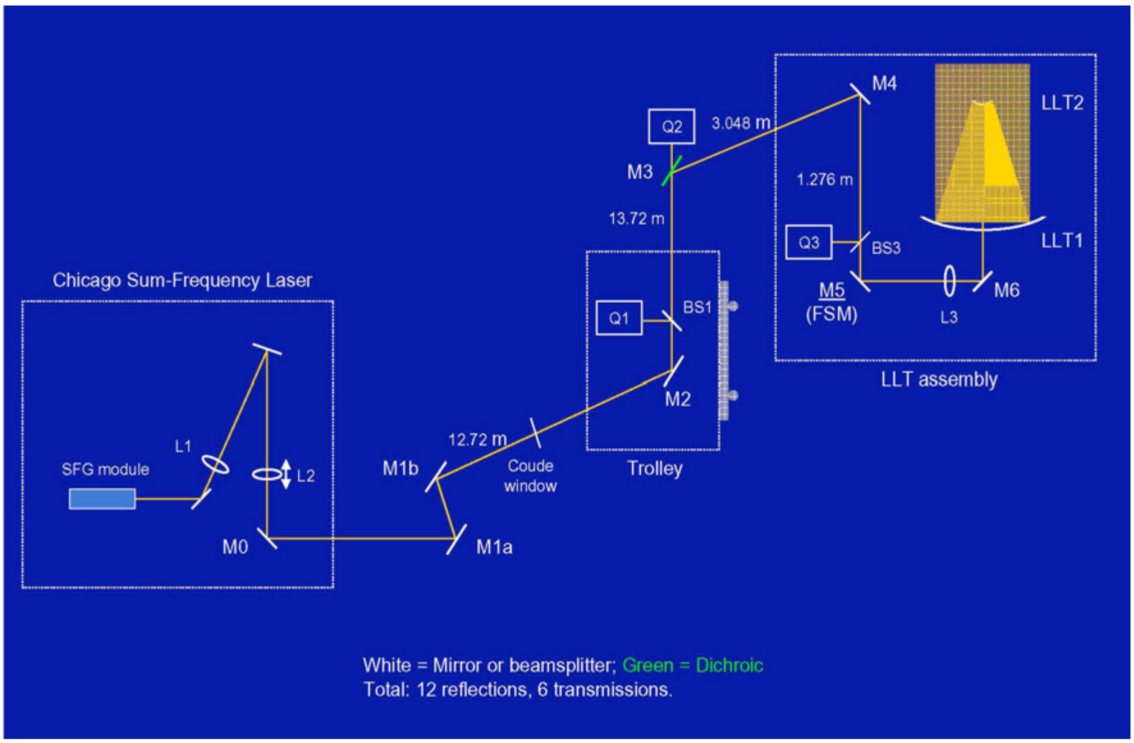

Figure 1. BTO Layout and Definitions

2. Conventions

The quad cells Q1, Q2, Q3 are assumed to be individually calibrated, with outputs converted to millimeters of spot deflection. These outputs are designated as q_1, q_2, q_3, with x and y components. Quad cell calibrations, defined as the on-axis centroid change per mm, are a function of spot size and axis, and will be discussed in Section 4.

The coordinate system used here follows the optical path (raytrace convention) and moves with the telescope. The x axis is parallel to the dec axis at all locations, and the y axis is perpendicular to it and the local beam direction. For example, at Q3, with the telescope at zenith, the x axis is east-west (horizontal) and the y axis is vertical. At Q2, the x axis is east-west, the y axis is north-south. As all of the BTO starting with M1b lies in a plane, the x direction will also be referred to as the out-of-plane direction.

The sign convention for x is that positive is west everywhere. The sign convention for y is that positive is

up from M1 to M2;
north from M2 to M3;
up from M3 to M4;
south from M4 to M5/FSM;
down at Q3;
down at Q1.

At Q2, which is mounted out of plane, y is north and x is up.

3. Input and Output Mapping

The relationship between the logical coordinate system above and the way the mirror actuators and the quad cell sensors are physically mounted is best described with pictures. In these pictures, any actuator that needs its polarity reversed in software is labeled “reversed”. For all actuators, an extension is a positive move, and contraction is negative. The exception is M2rot, whose polarity is shown in Fig. 3.

Quad cell x and y assignments are also shown in these pictures, while quad cell connections, calibrations and polarity swaps are discussed in the next section

Figure 2 shows the two-inch version of the M1b mirror, with its motors labeled x and y. By convention, the horizontal axis is x, connected to channel 1, the vertical is y, connected to channel 2. With the mounting shown, the y axis is reversed (actuator extension pushes the beam down) and x is non-reversed (actuator extension pushes the beam west).

The four-inch version M1b has a different lever arm (scaling), and may require sign reversals, depending on mounting details.

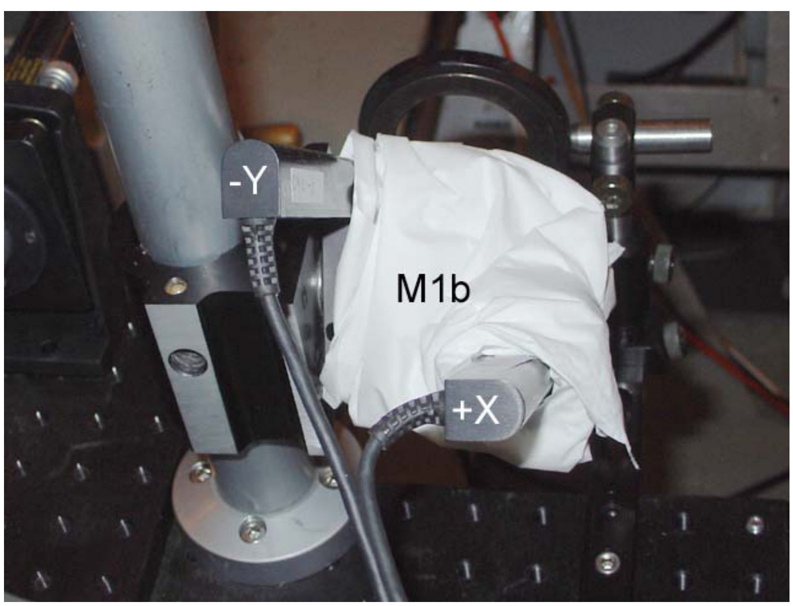

Figure 2. M1b Actuators

Figure 3 shows the trolley assembly, with quad cell Q1, and the M2 mirror with its two actuators M2x and M2y, and the M2rot rotary stage. Positive rotation of M2r is indicated by the arrow; this is a +y deflection. M2y creates –y deflection, additive to M2rot. M2x is non-reversed. M2x, M2y, and M2rot appear on channels 5, 6, 7, respectively.

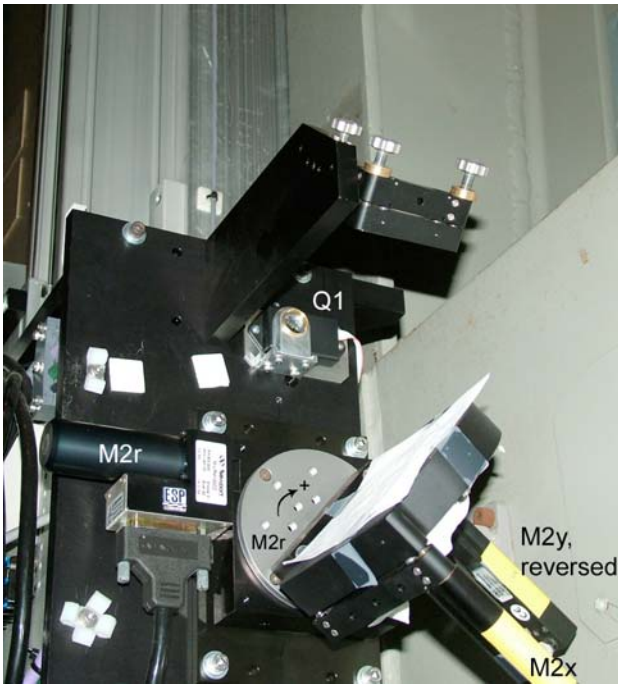

Figure 3. Trolley Assembly

Figure 4 shows the mounting arrangement of Q2 and M3. Both axes of M3 are sign-reversed. The x axis appears on channel 1, y on channel 2.

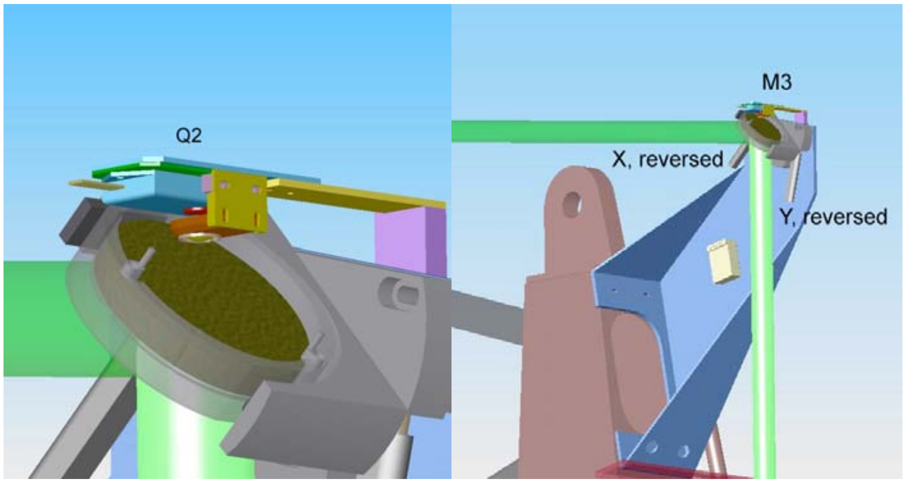

Figure 4. Q2 and M3

Figure 5 shows M4, Q3, the camera, iris, and fast steering mirror (FSM) on the side of the laser launch telescope (LLT).

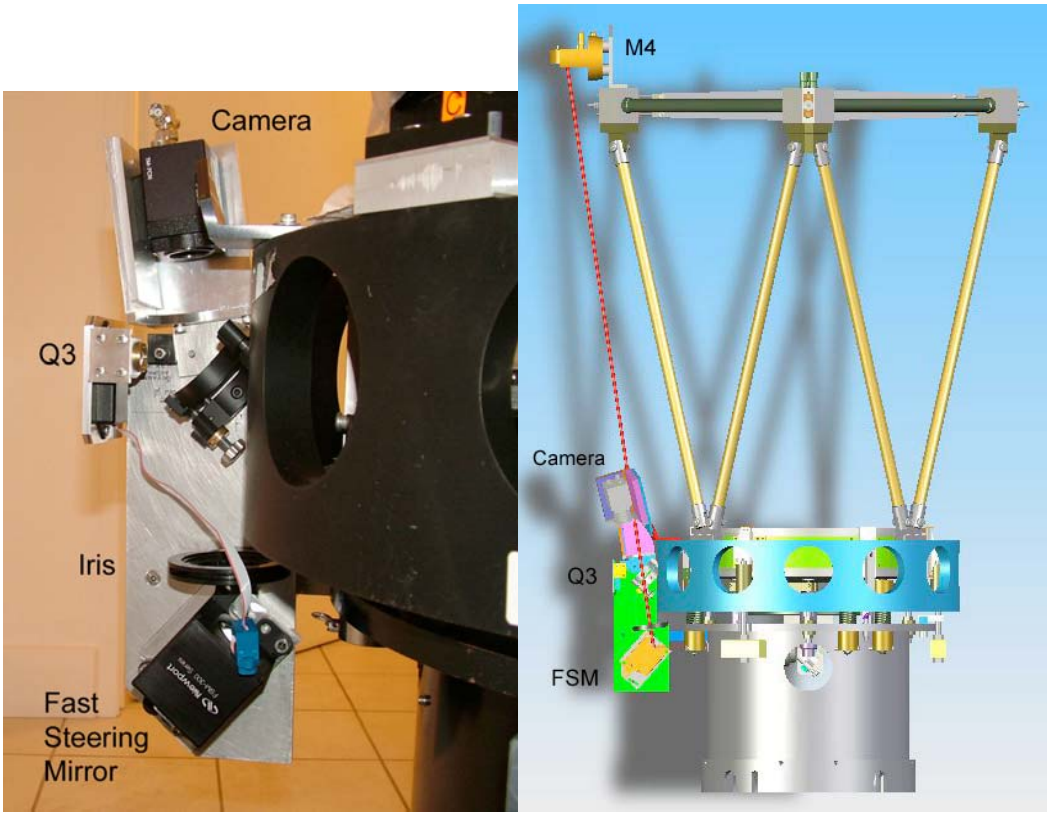

Figure 5. Laser Launch Telescope, with M4, Q3, FSM.

4. Quad cell Connections, Calibrations, Polarity

Figure 6 shows the orientation of the quad cells. Table 1 shows the quad cell connections, A/D channel assignments, and axis polarity.

Figure 6. Quad Cell Orientation and Polarity

The cabling from the quad cell 9-pin connector to the A/D terminal block, and the A/D channel sequence, are the same for the three quad cells. Any axis swaps and polarity changes are made in software. Table 1 summarizes the physical to logical mapping of the quad cells. A negative sign means the signal should be inverted in software. The black wire of the black/red pair is signal ground, and the black wire of the black/white pair connects to pin 6 of the quad cell 9-pin connector.

Table 1. Quad Cell Mappings

	Axis, channel # White, pin 2	Axis, channel # Black/white, pin 6	Sum signal channel # Red
Q1	-X, 00	-Y, 01	02
Q2	-Y, 03	-X, 04	05
Q3	-X, 08	-Y, 09	10

Quad cell calibrations are calculated using the assumptions that the laser beam is single-mode gaussian and has a waist at the LLT iris (basically, Q3) of 6.0 mm $1/e^2$ diameter. The spot sizes at Q1 and Q2 are not very different from that at Q3; a small correction for standard Gaussian beam propagation is done for zenith only, and the variation in spot size with declination is neglected. For Q1x and Q3y, the gain constant is lower because of the double reflection from their beamsplitters. Q2 does not have this effect because it looks at leakage through the M2 mirror. The details of this calculation are in the accompanying spread sheet “bto beam control v9.xls” The resulting x and y gain constants are listed in Table 2.

Table 2. Quad Cell Gain Constants

	X gain (mm^{-1})	Y gain (mm^{-1})
Q1	0.478	0.490
Q2	0.520	0.520
Q3	0.522	0.508

5. Partitioning the Control

The BTO optics at M1 and before are fixed to the ground in the coude room. So, for M1 only, define an x′, y′ which are rotated with respect to x, y by the hour angle of the telescope. On the optical path from M1 to M2, a desired optical tilt in x, y coordinates is then rotated into x′, y′ coordinates before calculating mechanical tilts to be commanded to M1.

By applying this coordinate transformation to M1 commands, the control problem then separates into non-interacting x and y parts. These can be analyzed as two independent three-sensor, three-actuator systems.

6. Mechanical and Optical Angles

Commands to the actuators are expressed two ways. The first is an actuator position in microns, with symbols m_1, m_2, m_3. The second is as an optical deviation in radians, with symbols a_1, a_2, a_3. For M2 and M3, the conversion between the two is:

$$m_x = a_x D_x / (2\cos(\theta))$$
$$m_y = a_y D_y / 2$$

Here, D is the lever arm of the actuator, and θ is the angle of incidence. It is assumed that the mirror tilt out of plane (x) is small, but that the mirror in-plane angle is arbitrary and has the value θ. The value of θ for M2 is (90 + dec) /2; for M3 it is (90+15.1)/2, or 52.55 degrees. For M1b using the 2-inch mount, the measured θ is (90-3.55)/2, or 43.22 degrees.

For M1b, the hour angle rotation must be included:

$$m_{1x} = \left(a_{1x}\cos(h) + a_{1y}\sin(h)\right)D_{1x} / (2\cos(\theta))$$
$$m_{1y} = \left(-a_{1x}\sin(h) + a_{1y}\cos(h)\right)D_{1y} / 2$$

Here h is the hour angle of the telescope, in radians

Table 2 list the lever arms of the various mounts. For all these mounts, the x and y lever arms are identical.

Table 3. Mirror Mount Lever Arms

	Mount type (Newport)	Lever arm (microns)
M1b, 2 inch	U200-AC	56100
M1b, 4 inch	605-4	51100
M2	U400-AC2X	107700
M3	605-4	51100

7. Interaction and Control Matrices

The interaction matrix is simple enough: M1 affects Q1, Q2 and Q3; M2 affects Q2 and Q3; and M3 affects Q3. Calling r_{ij} the distance from M_i to Q_j, the interaction matrix for either x or y can be written out as follows:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} r_{11} & 0 & 0 \\ r_{12} & r_{22} & 0 \\ r_{13} & r_{23} & r_{33} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

The inverse of this interaction matrix is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{r_{11}} & 0 & 0 \\ -\frac{r_{12}}{r_{11}r_{22}} & \frac{1}{r_{22}} & 0 \\ \frac{r_{12}r_{23} - r_{13}r_{22}}{r_{11}r_{22}r_{33}} & -\frac{r_{23}}{r_{22}r_{33}} & \frac{1}{r_{33}} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

As r_{11}, r_{22} and r_{33} are always strictly positive, the control matrix is never singular. The control law can be expressed algebraically as follows:

$$a_1 = \frac{q_1}{r_{11}}$$

$$a_2 = \frac{q_2}{r_{22}} - \frac{r_{12}}{r_{11}r_{22}} q_1$$

$$a_3 = \frac{q_3}{r_{33}} - \frac{r_{23}}{r_{22}r_{33}} q_2 + \frac{r_{12}r_{23} - r_{13}r_{22}}{r_{11}r_{22}r_{33}} q_1$$

A suggested scheme for BTO initialization and exception handling is as follows. Define a quad cell signal q_i as "valid" if its sum signal is above some threshold. The rule is then to set to zero each a_i in the above equations that has any invalid q as an input. As the a_i, after transformation to m_i, are the input to an integrator-type control law, this has the effect of freezing, or not updating, the relevant actuator. In practice, this constraint means that the first invalid q freezes its correspondingly numbered actuator, and all later actuators. Doing so should create a system which automatically initializes itself as well as possible, short of invoking some search procedure.

8. Spacings

Call the radial spacing from the 200-inch optic axis to the M2-M3 optic axis D. Viswa has a value of 3531.6 mm for D; independently and more roughly, I have measured 3455.0 mm. Viswa's value will be used here. Call the latitude of the 200-inch (+33.356 degrees) "lat", and telescope declination "dec". The calculations also assume that Q1 is at about the same location as M2, Q2 is at M3, Q3 is at M5/FSM.

With a combination of measurements and data from drawings, Viswa and I came up with the following spacings, at zenith.

M1-M2	12725 mm
M2-M3	13716 mm
M3-M4	3048 mm
M4-M5	1276 mm

This plus some geometry leads to the following values for the distances r_{ij} from mirror M_i to sensor Q_j. The units are mm.

$$r_{11} = 12725 + D\left(\frac{1}{\cos(lat)} - \frac{1}{\cos(dec)}\right) = 16963 - \frac{3531.6}{\cos(dec)}$$

$$r_{22} = 13716 - D(\tan(lat) - \tan(dec)) = 16041 + 3531.6\tan(dec)$$

$$r_{33} = 4324$$

$$r_{12} = r_{11} + r_{22}$$

$$r_{13} = r_{12} + r_{33}$$

$$r_{23} = r_{22} + r_{33}$$

All of these distances except r_{33} change with declination.

The accompanying spreadsheet "bto control v8.xls" calculates values of M1-M2 and M2-M3 distance, M2 angle of incidence, and M2 projected clear aperture for a range of zenith angles and declinations. Table 3 reprints the results. The North limit of +65.2 degrees declination is set by the bottom limit of trolley travel; this number should be updated when the trolley limit is precisely measured.

9. Finishing the Controller

The use of the above equations and numbers results in a set of actuator position errors in microns. To make a traditional integrate-only (PID minus the P and D) controller, these errors are multiplied by –k, where k is a dimensionless gain constant less than one, and used to increment the current commanded actuator positions. Software limits are then imposed on the resulting positions, then position commands are sent out. Adding amenities such as saving /restoring positions from file, loop on/off commands, background subtraction, piping data to a monitoring function, and the like, finish the controller.

Table 4. BTO Spacings and Angles vs Zenith Angle

zenith angle (deg)	dec (deg)	M1 to M2 (mm)	M2 to M3 (mm)	M2 angle of incidence (deg)	M2 projected clear aperture (mm)	
-45.00	-11.64	13347.29	10663.47	39.18	77.52	South limit
-40.00	-6.64	13397.61	10979.86	41.68	74.69	
-35.00	-1.64	13420.04	11289.87	44.18	71.72	
-33.36	0.00	13421.49	11391.23	45.00	70.71	Equator
-30.00	3.36	13415.43	11598.32	46.68	68.61	
-25.00	8.36	13383.60	11909.96	49.18	65.37	
-20.00	13.36	13323.32	12229.71	51.68	62.01	
-15.00	18.36	13232.16	12563.02	54.18	58.53	
-10.00	23.36	13106.28	12916.27	56.68	54.93	
-5.00	28.36	12939.97	13297.25	59.18	51.24	
0.00	33.36	12725.00	13716.00	61.68	47.44	Zenith
5.00	38.36	12449.48	14185.93	64.18	43.56	
10.00	43.36	12096.00	14725.76	66.68	39.59	
15.00	48.36	11638.42	15362.82	69.18	35.55	
20.00	53.36	11035.94	16138.91	71.68	31.44	
25.00	58.36	10221.62	17121.90	74.18	27.26	
30.00	63.36	9077.89	18430.16	76.68	23.04	
31.80	65.16	8547.19	19019.27	77.58	21.51	North limit