

From Olsen, Blum, & Rigaut (2003, AJ, 126, 452), the following expression relates the surface brightness of the environment, the distance modulus, the photometric error, and the stellar magnitude for a general luminosity function $\Phi(M)$, under the assumption that all of the photometric error is due to crowding:

$$\Sigma_m > 2M - 2.5 \log \left[\frac{4}{\pi} \left(\frac{\sigma_m}{1.086 a_{\text{res}}} \right)^2 \times \frac{\int_{M_{\text{lo}}}^{M_{\text{hi}}} 10^{-0.4M'} \Phi(M') dM'}{\int_{M_{\text{lo}}}^M 10^{-0.8M'} \Phi(M') dM'} \right] + (m - M)_0 \quad (1)$$

If we adopt a luminosity function $\phi(L) \propto L^\alpha$ (such that $\log_{10} \Phi(M) = -0.4(\alpha+1)M + \text{const.}$), then we get a clean analytical expression for the crowding-induced photometric error σ_m :

$$\sigma_m = 10^{-0.2(\Sigma_m - (m-M)_0 - 2M + M_{\text{lo}})} \left(\frac{1.086 a_{\text{res}}}{2} \sqrt{\pi \frac{(2 + \alpha)(10^{-0.4(3+\alpha)(M-M_{\text{lo}})} - 1)}{(3 + \alpha)(10^{-0.4(2+\alpha)(M_{\text{hi}}-M_{\text{lo}})} - 1)}} \right) \quad (2)$$

For stellar populations with ages $> \sim 3$ Gyr in the near-infrared, $\alpha = -1.84$ is a good approximation to a luminosity function drawn from a Salpeter IMF. In the plots below, I show the power-law luminosity function $\phi(L) \propto L^{-1.85}$ (black line) compared to luminosity functions with ages 1 Gyr (blue line), 5 Gyr (green line), and 10 Gyr (red line) and solar metallicity, as calculated from Padova isochrones. In the plot on the right I show the corresponding crowding-induced photometric errors vs. K magnitude, assuming $\Sigma_K = 18 \text{ mags arcsec}^{-2}$, $(m-M)_0 = 24.45$, and a diffraction-limited 10-m telescope. For the discrete ages, the

calculation was done numerically using equation (1), while for the power-law LF I used the purely analytical equation (2).

