

# Sensitivity of different wavefront sensing methods to measuring focus from the sodium laser guide star

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## 1. Introduction

This is a brief note describing the sensitivity of the different natural guide star (NGS) wavefront sensing options to the focus measurement from the sodium laser guide star (NaLGS). The sensitivity to focus is calculated using the sky coverage simulator. For a more detailed description of the sky coverage simulator, see Ref 1 (which is in the Docushare).

The three NGS wavefront sensor (WFS) options that we are investigating are:

- (TT) There are multiple NGS WFS measuring the tip and tilt modes only. For computational reasons, there are assumed to be as many NGS WFS as there are NGS in the field. This approximation yields estimates (in nm) about 20 percent more favourable than the best combination of three NGS.
- (TTFA) There is a single NGS WFS that measures the tip/tilt/focus/astigmatism modes only. If there is more than one NGS in the field, the best NGS is chosen by evaluating all the NGS and choosing the NGS that produces the lowest wavefront error.

- (TTR) There is a single NGS WFS that measures the tip/tilt modes, and a Rayleigh laser guide star (RLGS). If there is more than one NGS in the field, the best NGS is chosen by evaluating all the NGS and choosing the NGS that produces the lowest wavefront error.

Section 2 outlines the theory of the sensitivity calculations, and the results are presented in Section 3.

## 2. Some theory

We assume the atmosphere consists of a finite number of layers of turbulence. The phase screen at each turbulent layer is modelled as the sum of Zernike polynomials, with coefficients  $a(i)$ . We use the first six radial orders of Zernikes. The Zernikes at each layer are projected to the aperture with the transformation matrix  $T$  such that

$$b = Ta(i). \quad (1)$$

The  $T$  matrix takes into account the off-axis effects of the NaLGS, NGS and RLGS, and finite height of the NaLGS and RLGS, and the summation over the atmospheric layers.  $T_L$  is the transformation matrix for the NaLGS,  $T_n$  is the transformation matrix for the NGS,  $T_R$  is the transformation matrix for the RLGS, and  $T_s$  is the transformation matrix for the science points.

We estimate the wavefront,  $\hat{a}$ , with a minimum variance estimator,  $E$ , from the measurements from the NaLGS  $b_L$ , NGS  $b_n$ , and RLGS  $b_R$ , such that

$$\hat{a} = Eb_{GS}, \quad (2)$$

where

$$b_{GS} = \begin{pmatrix} b_L \\ b_n \\ b_R \end{pmatrix}. \quad (3)$$

The minimum variance estimator,  $E$ , is given by

$$E = CT_{GS}^T(T_{GS}CT_{GS}^T)^\dagger \quad (4)$$

where  $C$  is the covariance of the atmosphere, and  $T_{GS}$  is the matrix of all the guide star projection matrices

$$T_{GS} = \begin{pmatrix} T_L \\ T_n \\ T_R \end{pmatrix}. \quad (5)$$

The sensitivity,  $S$ , of the Zernike modes at each science point is given by

$$S = T_s E \alpha, \quad (6)$$

where  $\alpha$  is 1 for the focus modes of the NaLGS, and 0 otherwise,

$$\alpha = \begin{cases} 1, & \text{Focus mode of NaLGS} \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The wavefront error,  $\sigma^2$  (rad<sup>2</sup>), arising from 1 radian of error in measuring the focus mode from the NaLGS is

$$\sigma^2 = T_s^T W T_s E \alpha \alpha^T E^T \quad (8)$$

$$= S W S^T. \quad (9)$$

In the next section, we will plot the sensitivity,  $S$ , of each Zernike mode at each point in the science field for each of the three WFS methods, and plot the wavefront error,  $\sigma^2$ , as a function of the position of the NGS in the field for the three WFS methods.

### 3. Some Results

It should be noted that there is no noise on the NGS measurements. We use the Cerro Pachon turbulence profile, an outer scale of 30m, and the standard 7 NaLGS asterism which is shown in Fig. 1. There is 1 DM conjugate to the ground layer, the NaLGS is at 90km and the RLGS at 20km. The RLGS is on axis.

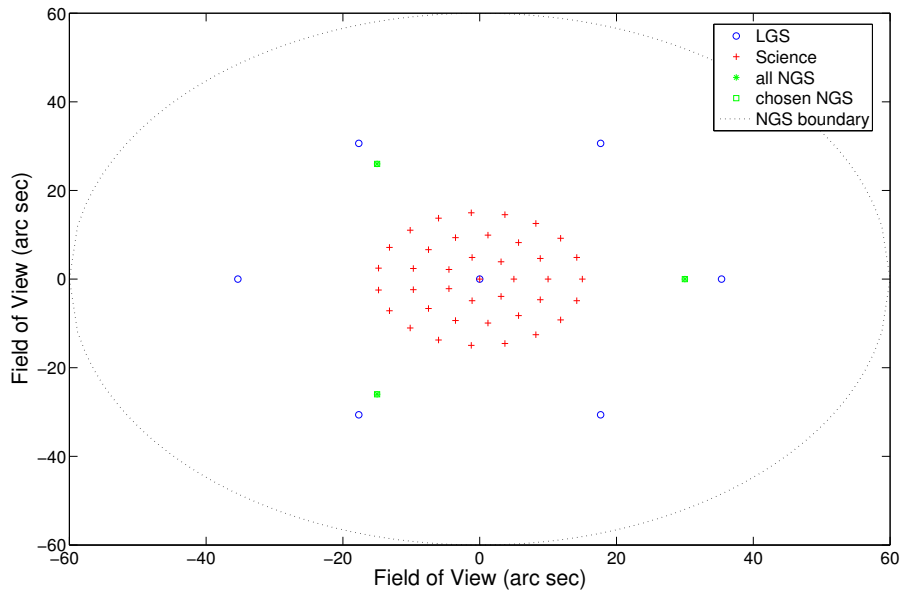


Fig. 1. The field of view showing the science evaluation points (pluses), NGS (squares), and NaLGS (circles).

There are 40 science points as shown by pluses in Fig 1. The first point is on-axis, the next seven are equally spaced on a ring of radius 5 arc sec, the next 13 are equally spaced on a ring of radius 10 arc sec, and the last 21 science points are equally spaced on a ring of radius 15 arc sec.

The sensitivity,  $S$ , of the 27 Zernike modes at each of the 40 science points for the TT WFS is shown in Fig. 2. Three NGS arranged in a equilateral triangle centred on-axis and with side length 30 arc sec have been used for this figure.

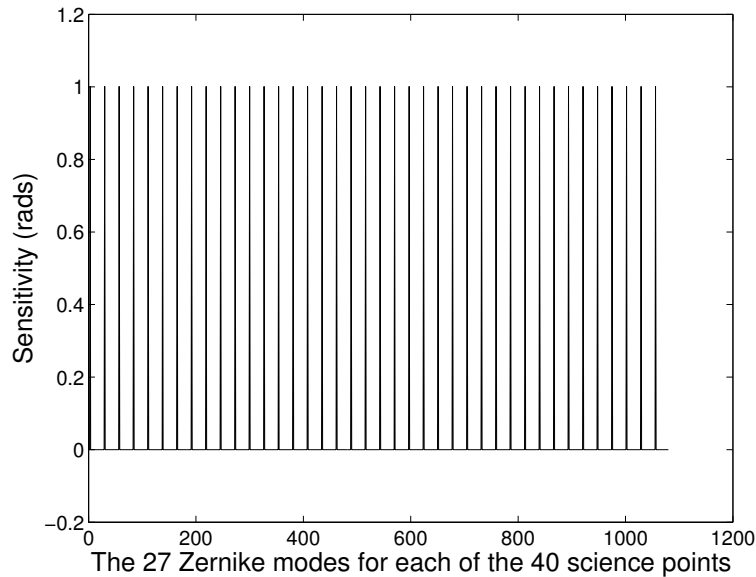


Fig. 2. The sensitivity,  $S$  (rads), of the first 27 Zernike modes for the 40 science points when measuring tip and tilt from the NGS.

Fig. 2 has peaks at points 3, 30, 57 ... , which corresponds to the focus modes at each science point. All other modes are zero for all science points, showing that an error in measuring the focus from the NaLGS does not affect the estimation of any other mode with multiple TT NGS. The focus modes have a value of 1.00, showing

that an input error of 1 rad of focus on the NaLGS measurement creates 1 rad of error in focus in the wavefront estimate.

The sensitivity  $S$  of the 27 Zernike modes at each of the 40 science points for the TTFA WFS is shown in Fig. 3. One NGS on-axis was used to generate this figure.

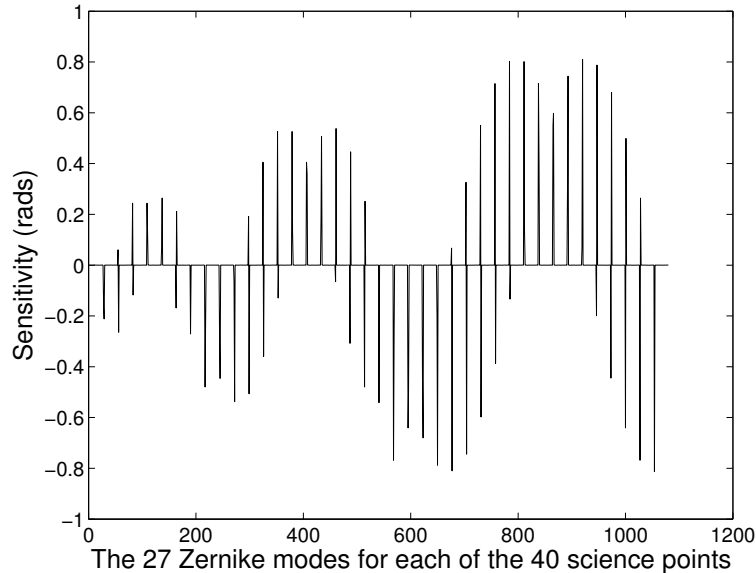


Fig. 3. The sensitivity,  $S$  (rads), of the first 27 Zernike modes for the 40 science points when measuring tip/tilt/focus/astigmatism from the NGS. The NGS is on-axis.

For the TTFA WFS, the only modes that are sensitive to the focus from the NaLGS are the tip and tilt modes. This is due to an error in measuring tilt anisoplanatism. The error in measuring tilt anisoplanatism is a function of the position of the science point in the field, with science points in the outer ring having the largest sensitivity. With the NGS on-axis, 1 rad of focus error corresponds to, for the least favourable science point, 0.8 rads of error in the estimate of tilt anisoplanatism.

The sensitivity of the 27 Zernike modes at each of the 40 science points for the TTR WFS is shown in Fig. 4. One NGS on-axis was used to generate this figure.

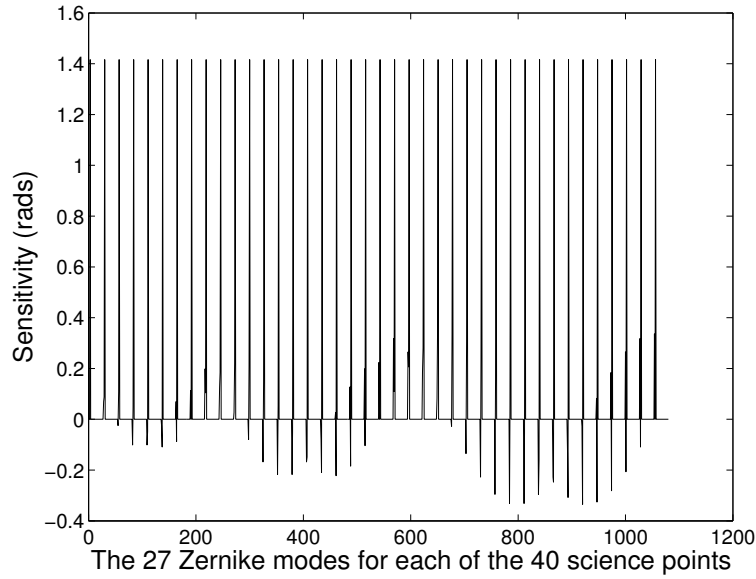


Fig. 4. The sensitivity,  $S$  (rads), of the first 27 Zernike modes for the 40 science points when measuring tip and tilt from the NGS, and there is a Rayleigh beacon. The NGS is on-axis

Fig. 4 shows that a TTR WFS produces both an error in the focus modes and an error in estimating the tilt anisoplanatism.

Fig.'s 3 and 4 were generated with a NGS on-axis. As the NGS is moved off-axis for the TTFA NGS WFS and TTR WFS, the amount of tilt anisoplanatism increases. This can be seen by comparing Fig. 5 for a NGS 30 arc sec off-axis with the on-axis case of Fig. 3.

Fig. 6 shows the error,  $\sigma^2$ , computed from Eq. (8), for the three WFS methods as a function of the position of the NGS. The TT WFS is insensitive to the position of

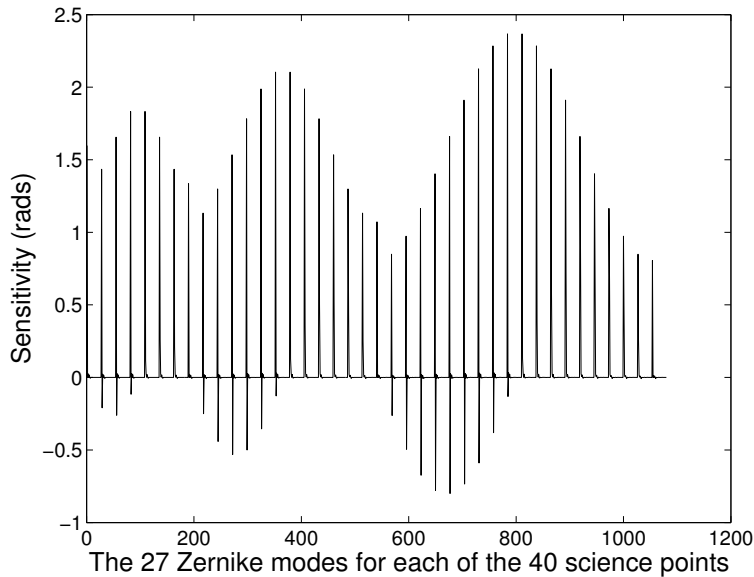


Fig. 5. The sensitivity,  $S$  (rads), of the first 27 Zernike modes for the 40 science points when measuring tip/tilt/focus/astigmatism from the NGS. The NGS is 30 arc sec off-axis.

the NGS, since the tilt anisoplanatism can be estimated with 3 NGS in a equilateral triangle, regardless of the side length of the triangle. For both the TTFA and TTR options, the wavefront error due to an error in the focus measurement from the NaLGS increases as the single NGS is moved off-axis. This effect is most severe with a TTFA WFS.

We see from Fig. 6 that a TTFA WFS is less sensitive to the focus measurement from the NaLGS than a TT WFS provided there is a NGS within 15 arc sec of on-axis. The probability of there being an NGS within a 15 arc sec radius in J band at the North Galactic Pole is 62 percent. So there is a 62 percent chance that the TTFA WFS will be less sensitive to focus from the NaLGS than the TT WFS. For



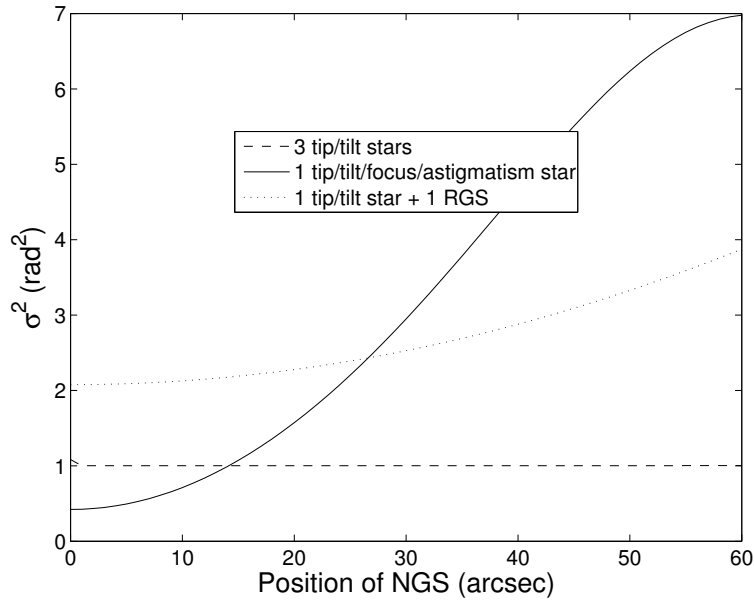


Fig. 6. The wavefront error,  $\sigma^2$  (rad<sup>2</sup>), averaged over the 40 science points resulting from an error in measuring the focus mode from the NaLGS.

comparison, the probability of an NGS within a 20 arc sec circle is 81 percent, and 91 percent within a 25 arc sec circle.

#### 4. Conclusion

In summary, the three wavefront sensing methods we are considering have different sensitivities to the focus measurement from the NaLGS. For multiple tip/tilt NGS WFS, an error in measuring the focus mode from the NaLGS produces a focus error on the estimated wavefront but no error in the other wavefront modes (no error in estimating the tilt anisoplanatism arises from the focus error).

For a single tip/tilt/focus/astigmatism NGS, there is no focus error in the estimated wavefront from an error in the measurement of the NaLGS, since the focus can

be measured from the NGS. However, the error in measuring the focus mode from the NaLGS produces an error in estimating tilt anisoplanatism. The error in tilt anisoplanatism is dependent on the position of the NGS and the position of the science point. The error gets worse as the NGS is further off-axis, and also for the science points further from the centre of the field.

For a single tip/tilt NGS WFS used in conjunction with a RLGS, an error in measuring the focus mode from the NaLGS produces both an error in focus in the wavefront estimate and an error in estimating the tilt anisoplanatism. The error in estimating the tilt anisoplanatism is dependent on the position of the NGS in the field.

## References

1. R. M. Clare and B. L. Ellerbroek, "Sky Coverage estimates for adaptive optics systems from computations in Zernike space," J. Opt. Soc. Am. A (submitted).