# On the possibility of using J and H and K bands for NGS WFS for NFIRAOS

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The NFIRAOS CoDR identified that sky coverage could be improved by using J and H, or J, H and K bands, rather than just J band for the near IR natural guide star (NGS) wavefront sensors (WFS). Using H, or H+K bands in addition to J band creates more photo-detcection events, but produces more background events. The Strehl is higher across the field in H and K bands than J, but the spot size increases at the larger wavelengths. Here, a comparison of the noise equivalent angle (NEA) $\sigma_\theta$ for the three options (J, J+H, J+H+K) is made to estimate the combination of these effects. No consideration of detectors is made; 10 electrons of read noise per pixel per read-out is assumed for all bands.

The NEA or tip/tilt measurement error $\sigma_\theta$ is on each subaperture (alternatively the noise equivalent angle) is given by

$$\sigma_\theta = \frac{\theta_B}{\mathrm{SNR}}, \tag{1}$$

where $\theta_B$ is the effective spot size of the subaperture guide star image, and SNR is the signal-to-noise ratio of a single subaperture. For a quadrant detector, the SNR is given by

$$\mathrm{SNR} = \frac{N_p}{\sqrt{N_p + 4N_b + 4\sigma_e^2}}, \tag{2}$$

where $N_p$ is the number of photodetection events per subaperture, $N_b$ is the number of background photodetection events per subaperture, and $\sigma_e$ is the rms detector read noise per pixel. When multiple bands are considered, eg J+H, the photo-detection events in those bands are added, as are the background events.

In the infra-red, we assume that the NGS images contain a diffraction-limited core, and the effective spot size is given by[1]

$$\theta_B = \frac{1}{4\int_0^\infty I(0,\kappa)\mathrm{d}\kappa}, \tag{3}$$

where $I(0,\kappa)$ is the OTF. Expanding and simplifying[2] gives the spot sizes for J, J+H, and J+H+K respectively

$$\theta_{B,J} = \frac{3\pi\lambda_J\sqrt{N_{sa}}}{16D} \tag{4}$$

$$\theta_{B,JH} = \frac{3\pi\sqrt{N_{sa}}(N_{p,J} + +N_{p,H})}{16D(\frac{N_{p,J}}{\lambda_J} + \frac{N_{p,H}}{\lambda_H})} \tag{5}$$

$$\theta_{B,JHK} = \frac{3\pi\sqrt{N_{sa}}(N_{p,J} + N_{p,H} + N_{p,K})}{16D(\frac{N_{p,J}}{\lambda_J} + \frac{N_{p,H}}{\lambda_H} + \frac{N_{p,K}}{\lambda_K})} \tag{6}$$

where $N_{sa}$ is the total number of subapertures for the NGS WFS.

The number of photons per subaperture in the J, H, and K bands respectively are given by

$$N_{p,J} = \frac{z_J 10^{-m_J/2.5}\chi A_T t_s S_J}{N_{sa}} \tag{7}$$

$$N_{p,H} = \frac{z_H 10^{-m_H/2.5}\chi A_T t_s S_H}{N_{sa}} \tag{8}$$

$$N_{p,K} = \frac{z_K 10^{-m_K/2.5}\chi A_T t_s S_K}{N_{sa}}, \tag{9}$$

where $z$ is the intensity of a zero magnitude star, $m$ is the magnitude of the NGS, $\chi$ is the end-to-end efficiency of the optics and WFS detectors, $t_s$ is the integration time, $S$ is the Strehl ratio, and $A_T$ is the area of the telescope pupil ($\pi(D/2)^2$).

The number of background photodetection events per pixel in each subaperture is given by

$$N_{b,J} = \frac{z_{b,J}\chi A_T t_s w^2}{N_{sa}} \tag{10}$$

$$N_{b,H} = \frac{z_{b,H}\chi A_T t_s w^2}{N_{sa}} \tag{11}$$

$$N_{b,K} = \frac{z_{b,K}\chi A_T t_s w^2}{N_{sa}}, \tag{12}$$

where $z_b$ is the background intensity per square arc second in the plane of the telescope, and $w$ is the angular subtense of the square pixel.

The Strehl ratios, $S$, at different points in the field were calculated in LAOS for different wavelengths, and are plotted in Fig. 1. The improvement in Strehl at longer wavelengths is most noticeable at the edge of the field: Strehl at 60 arc sec is 0.04 in J, and 0.16 in H, and 0.35 in K. These Strehls are calculated for the baseline NFIRAOS design.

In order to work out the magnitude in H and K of a star of a given magnitude in J, we use the J-H and J-K color magnitude tables in Ref 3. Without knowing the relative probabilities of the different spectral types, I choose K0 dwarf as the an example spectral class, which sits in the middle of the tables for J-H and J-K magnitudes. For the K0 dwarf, J-H=0.45, and J-K=0.53.

In Fig. 2, the NEA, $\sigma_\theta$, is plotted for J, J+H, and J+H+K bands for a K0 dwarf for a 100 Hz sampling rate and 10 electrons of read noise per pixel per read-out for an on-axis star in (a), and at 1 arc min in (b), versus the J magnitude. Fig. 2(a) shows a gain of between 0.5 and 1 magnitudes in going from J to J+H on-axis, but little extra gain in going to J+H+K on-axis. However, at 1 arc min, there is a further half

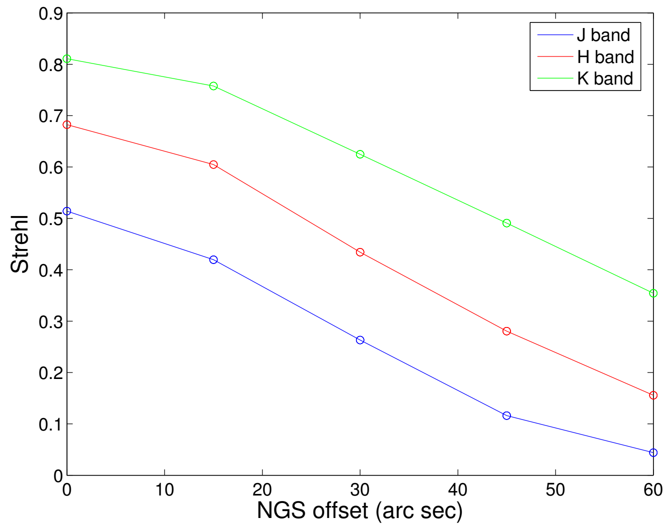
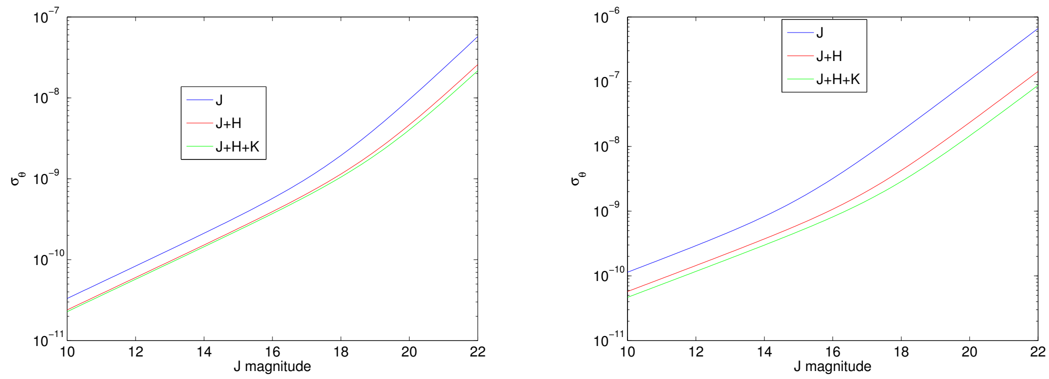



Fig. 1. Strehl ratio at different points in the field for J, H and K bands.

magnitude gain in going to K band.

In Fig. 3, the reduction in the NEA from using J+H versus J only, ie $\sigma_\theta(J)/\sigma_\theta(J+H)$), and J+H+K versus J only, ie $\sigma_\theta(J)/\sigma_\theta(J+H+K)$) is plotted versus the J magnitude for three points in the field. The relative gain in going to J+H, or J+H+K, is greatest at the edge of the field, which is due to the larger Strehl improvement at longer wavelengths at the edge of the field, and for dimmer stars, where we are read noise rather than photon limited.

In summary, using J and H bands, instead of just J band, can reduce the NEA by a factor of 1.5 on-axis and up to 4.5 at the edge of the field. There is little further improvement in including K band also when on-axis, but the improvement at the edge of the field with K increases to 7.5. In order to include H and K bands in the



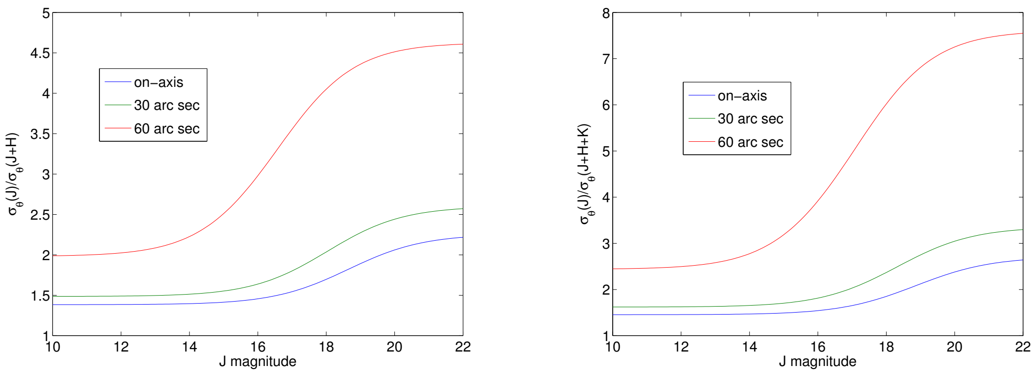

(a) (b)

Fig. 2. Noise Equivalent Angle, $\sigma_\theta$, for J, J+H, and J+H+K for a K0 dwarf (a) on-axis and (b) at 1 arc min.

sky coverage simulator, and hence work out the improvement in tip/tilt wavefront error, it is necessary to estimate the relative probabilities of different spectral classes at each J magnitude. The improvement in Strehl at the edge of the field in H opens the question of whether a 2 arc min diameter field, which was optimal for J band only, is optimal if H or H+K are used.

## References


1. B. L. Ellerbroek and D. W. Tyler, "Adaptive optics sky coverage calculations for the Gemini-North telescope," Pub. Ast. Soc. Pac. **110**, 165-185 (1998).
2. G. A. Tyler and D. L. Fried, "Image-position error associated with a quadrant detector," J. Opt. Soc. Am. **72**, 804-808 (1982).




(a) (b)

Fig. 3. Relative reductuion in NEA in going from J to (a) J+H, and (b) J+H+K, for three positions in the field.


3. M. L. Bessell and J. M. Brett, "JHKLM photometry: standard systems, passbands, and intrinsic colors," Pub. Ast. Soc. Pac. **100**, 1134-1151 (1988).

4. www.gemini.edu/sciops/instruments

Table 1. Parameters used for the calculations. Data values are from Ref. 4.

| Parameter | Symbol | Value |
|---|---|---|
| Telescope diameter | $D$ | 30 m |
| Pixel subtense | $w$ | $\lambda_J/D$ rads = 8.6mas |
| Number of subapertures | $N_{sa}$ | 1 |
| Integration time | $T_s$ | 0.01 s |
| Read noise | $\sigma_e$ | 10 e /pixel/readout |
| End-to-end efficiency of optics | $\chi$ | 0.4 |
| Wavelength in J | $\lambda_J$ | 1.25$\mu$m |
| Wavelength in H | $\lambda_H$ | 1.65$\mu$m |
| Wavelength in K | $\lambda_K$ | 2.2$\mu$m |
| Background intensity in J | $z_{b,J}$ | 1191 photons $m^{-2}$ arc sec$^{-2}$ $s^{-1}$ |
| Background intensity in H | $z_{b,H}$ | 5505 photons $m^{-2}$ arc sec$^{-2}$ $s^{-1}$ |
| Background intensity in K | $z_{b,K}$ | 9395 photons $m^{-2}$ arc sec$^{-2}$ $s^{-1}$ |
| Intensity of $m$=0 star in J | $z_J$ | 3.76 $\times$ $10^9$ photons $m^{-2}$ $s^{-1}$ |
| Intensity of $m$=0 star in H | $z_H$ | 3.17 $\times$ $10^9$ photons $m^{-2}$ $s^{-1}$ |
| Intensity of $m$=0 star in K | $z_K$ | 1.30 $\times$ $10^9$ photons $m^{-2}$ $s^{-1}$ |