Analytical evaluations of closed-loop adaptive optics spatial power spectral densities

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1 Introduction

This note rederives and corrects some errors in the analytical expressions for the spatial frequency power spectral densities (PSDs) of servo-lag, noise and spatial aliasing, as previously given in [2, 1]. The work in [2] was based on and a development of the initial studies presented in [4]. The current results may be compared and contrasted to those in [3].

2 Preliminaries

These results are required prior to evaluating the PSDs in section 3.

2.1 Operators

The operator formalism used in the derivations are reviewed here. For a mean-gradient type wavefront sensor (WFS), with rectangular sub-apertures in a regular rectangular grid, the measurement operator $\mathcal{M}$ is (see [2]):

$$\mathcal{M} = \mathrm{comb} \times [\Pi * \nabla], \tag{1}$$

which has the Fourier transform (denoted interchangably by tilde $^\sim$ or by $\mathcal{F}$)

$$\mathcal{F}[\mathcal{M}] = \mathrm{comb} * [\widetilde{\Pi} \times \widetilde{\nabla}], \tag{2}$$

since the comb function is its own Fourier transform (see below). The corresponding functions (and their abbreviated notations) are:

$$\Pi\left(\frac{\mathbf{x}}{d}\right) = \begin{cases} 1, & |x| \leq d, |y| \leq d \\ 0, & \text{otherwise} \end{cases}, \tag{3}$$

$$\nabla = \frac{\partial}{\partial \mathbf{x}} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right], \tag{4}$$

$$\mathrm{comb}\left(\frac{\mathbf{x}}{d}\right) = \sum_{\mathbf{m}} \delta\left(\frac{\mathbf{x}}{d} - \mathbf{m}\right) = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \delta\left(\frac{x}{d} - m\right) \delta\left(\frac{y}{d} - n\right), \tag{5}$$

where boldface denotes a (2-element) vector. The spatial plane coordinate is $\mathbf{x} = (x, y)$, with the Fourier conjugate spatial frequency variable $\mathbf{f} = (f_x, f_y)$. The Fourier transforms of (3)-(5) are:

$$\widetilde{\Pi}(\mathbf{f}d) = \mathrm{sinc}(\mathbf{f}d) = \frac{sin(\pi f_x d)}{\pi f_x d} \times \frac{sin(\pi f_y d)}{\pi f_y d}, \tag{6}$$

$$\widetilde{\nabla} = 2\pi i \mathbf{f} = 2\pi i [f_x, f_y], \tag{7}$$

$$\mathcal{F}[\mathrm{comb}](\mathbf{f}d) = \sum_{\mathbf{m}} \delta(\mathbf{f}d - \mathbf{m}). \tag{8}$$

A wavefront reconstructor may be defined as the operator $\mathcal{R}$ that fulfills $\mathcal{R}[\mathcal{M}(\varphi_{\parallel})] = \varphi_{\parallel}$, where $\varphi = \varphi_{\parallel} + \varphi_{\perp}$ is the optical phase of the aberrated wavefront split into a low spatial frequency part $\varphi_{\parallel}$ defined by the band-filter domain of the AO system, and a high frequency part $\varphi_{\perp}$ that passes through the AO system unattenuated (but gives rise to fitting and aliasing errors). The Fourier domain recostructor that does this is

$$\widetilde{\mathcal{R}} = \frac{\mathbf{f}^{-1}}{4\pi i\,\mathrm{sinc}(\mathbf{f}d)}. \tag{9}$$

2.2 Closed loop

A simple model of closed loop operation is derived here. For a simple integrator with a fixed gain g, the mirror shape $s(\mathbf{x}, t)$ at any time step nt_i is computed as (omitting an additional time delay t_d and the spatial variable $\mathbf{x}$ for brevity)

$$s(t - nt_i) = s[t - (n+1)t_i] + g\widehat{\varphi}[t - (n+1)t_i], \tag{10}$$

where t_i is the WFS integration time, and $\widehat{\varphi}$ is the AO estimated residual wavefront error. To simplify even more we will use the abbreviated notation

$$s_n = s_{n+1} + g\widehat{\varphi}_{n+1}, \tag{11}$$

which allows us to write compactly $\widehat{\varphi}_n = G_n - s_n$. The function G_n is here the open-loop reconstruction of the wavefront plus reconstructed noise, i.e:

$$G_n = \mathcal{R}[\mathcal{M}(I_n) + \nu], \tag{12}$$

where $\nu(\mathbf{x}, t)$ is the WFS noise function, and I_n is defined below in (18). From the recursive relation (11) it is easy to show that (cf. [2]) the present mirror shape s_0 after N time steps is given by

$$s_0 = (1-g)^N s_N + g\sum_{n=1}^{N} G_n(1-g)^{n-1}. \tag{13}$$

If the AO system is operating in a steady state far enough away from the startup phase, so that initial conditions no longer matter, we can for any $0 < g < 1$ approximate $N \approx \infty$, which gives

$$s_0 = g\sum_{n=1}^{\infty} G_n(1-g)^{n-1}. \tag{14}$$

G_n will later be assigned different functions to represent different AO errors.

2.3 Taylor hypothesis

The method assumes the Taylor hypothesis of N_l discrete turbulence layers with frozen flow. Ignoring anisoplanatism (i.e. doing all calculations for a fixed θ) and introducing the wind velocity profile $\mathbf{v}_l$, we obtain the phase summed over layers after an arbitrary time delay τ as

$$\varphi(\mathbf{x}, t) = \sum_{l=1}^{N_l} \varphi_l(\mathbf{x} - \mathbf{v}_l\tau, t), \tag{15}$$

which has the Fourier transform

$$\widetilde{\varphi}(\mathbf{f}, t) = \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f}, t)\exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l\tau). \tag{16}$$

It is assumed that each layer φ_l follows von Karman turbulence statistics independently and with separate power, i.e.

$$\langle \widetilde{\varphi}_l{}^{\dagger}\widetilde{\varphi}_l\rangle = \frac{0.023}{r_{0l}^{5/3}}(f^2 + f_0^2)^{-11/6} \tag{17}$$

where $f_0 = 1/L_0$ and L_0 is the turbulence model outer scale, and r_{0l} is the Fried parameter per layer l.

2.4 WFS temporal integration

The measurement operator in (1) only represents the spatial wavefront sampling. To account for the finite integration time t_i of the WFS we must evaluate several (similar) integrals of the type

$$\widetilde{I}_n(\mathbf{f},t) = \frac{1}{t_i}\int_{-t_i/2}^{+t_i/2} d\tau\, \widetilde{\varphi}(\mathbf{f}, t - t_d - nt_i - \tau), \tag{18}$$

where t_d is an additional temporal delay due to e.g. CCD read-out, centroiding and reconstruction computations. The integer index n was included for generality and to show the relation to the closed loop function G_n, but for the remainder of this section we set $n = 0$ for brevity. Using the result (16) gives

$$\widetilde{I}_0(\mathbf{f},t) = \frac{1}{t_i}\sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f},t)\exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l t_d) \underbrace{\int_{-t_i/2}^{+t_i/2} d\tau\, \exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l\tau)}_{I'}. \tag{19}$$

The remaining integral I' can be evaluated to

$$\begin{aligned} I' &= \frac{1}{2\pi i\mathbf{f}\cdot\mathbf{v}_l}\left[\exp(\pi i\mathbf{f}\cdot\mathbf{v}_l t_i) - \exp(-\pi i\mathbf{f}\cdot\mathbf{v}_l t_i)\right] & (20)\\ &= \frac{\sin(\pi\mathbf{f}\cdot\mathbf{v}_l t_i)}{\pi\mathbf{f}\cdot\mathbf{v}_l} = t_i\,\mathrm{sinc}(\mathbf{f}\cdot\mathbf{v}_l t_i), & (21) \end{aligned}$$

and the whole expression becomes

$$\widetilde{I}_0(\mathbf{f},t) = \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f},t)\,\mathrm{sinc}(\mathbf{f}\cdot\mathbf{v}_l t_i)\exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l t_d). \tag{22}$$

3 Power spectral densities

3.1 Servo-lag

In this section $\varphi = \varphi_{\|}$. The closed-loop expression for the servo-lag error PSD Φ_{sl} is

$$\Phi_{sl}(\mathbf{f}) = \left\langle \left|\mathcal{F}\left\{\varphi(\mathbf{x},t) - s_0(\mathbf{x},t)\right\}\right|^2 \right\rangle. \tag{23}$$

Shifting the noise term to the noise PSD (to not count it twice), we have that $G_n = \mathcal{R}[\mathcal{M}(I_n)]$, cf. equations (14) and (22), which simplifies to $G_n = I_n$ upon invoking perfect reconstruction $\mathcal{R}[\mathcal{M}(\varphi_{\|})] = \varphi_{\|}$. Applying the Fourier transform and writing it out gives

$$\begin{aligned} \Phi_{sl}(\mathbf{f}) &= \left\langle \left| \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f},t) - \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f},t)\,\mathrm{sinc}(\mathbf{f}\cdot\mathbf{v}_l t_i)\, g\sum_{n=1}^{\infty}(1-g)^{n-1}\exp[2\pi i\mathbf{f}\cdot\mathbf{v}_l(t_d + nt_i)] \right|^2 \right\rangle & (24)\\ &= \left\langle \left| \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f},t)\left[1 - \mathrm{sinc}(\mathbf{f}\cdot\mathbf{v}_l t_i)\, g\sum_{n=1}^{\infty}(1-g)^{n-1}\exp[2\pi i\mathbf{f}\cdot\mathbf{v}_l(t_d + nt_i)]\right] \right|^2 \right\rangle & (25)\\ &= \sum_{l=1}^{N_l}\sum_{k=1}^{N_l} \left\langle \widetilde{\varphi}_l^{\dagger}(\mathbf{f},t)\widetilde{\varphi}_k(\mathbf{f},t)\right\rangle \Gamma_l^{\dagger}(\mathbf{f})\Gamma_k(\mathbf{f}), & (26) \end{aligned}$$

where we defined

$$\Gamma_l(\mathbf{f}) = 1 - \mathrm{sinc}(\mathbf{f}\cdot\mathbf{v}_l t_i)\exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l t_d)\, g\sum_{n=1}^{\infty}(1-g)^{n-1}\exp(2\pi i\mathbf{f}\cdot\mathbf{v}_l n t_i). \tag{27}$$

Separate turbulence layers are assumed to be uncorrelated, i.e. $\langle \varphi_l \varphi_k \rangle = \langle |\varphi_l|^2 \rangle \delta_{kl}$, so this simplifies to the tubulence PSD (17) and removes one summation. Defining $a = 1 - g$ and $b_l = 2\pi \mathbf{f} \cdot \mathbf{v}_l t_i$, the summation over n in expression (27) can be written

$$S_l = a^{-1} \sum_{n=1}^{\infty} a^n e^{inb_l} \tag{28}$$

$$= a^{-1} \sum_{n=1}^{\infty} a^n (\cos nb_l + i \sin nb_l), \tag{29}$$

This can be evaluated as two separate trigonometric series that have closed analytical forms:

$$\sum_{k=0}^{n} r^k \cos kx = \frac{(1 - r\cos x)(1 - r^n \cos nx) + r^{n+1} \sin x \sin nx}{1 - 2r\cos x + r^2}, \tag{30}$$

$$\sum_{k=1}^{n} r^k \sin kx = \frac{r \sin x(1 - r^n \cos nx) - (1 - r\cos x) r^n \sin nx}{1 - 2r\cos x + r^2} \tag{31}$$

These sums will convege as $n \to \infty$ for any $|r| < 1$. Evaluating the asymptotic forms, substituting back into (29) and combining terms gives eventually

$$S_l = \frac{e^{ib_l} - a}{1 - 2a\cos b_l + a^2}. \tag{32}$$

We can now jump to the final form of the PSD directly:

$$\Phi_{sl}(\mathbf{f}) = \frac{0.023}{(f^2 + f_0^2)} \times \sum_{l=1}^{N_l} r_{0l}^{-5/3} \left|\Gamma_l(\mathbf{f})\right|^2, \tag{33}$$

where

$$\Gamma_l(\mathbf{f}) = 1 - \mathrm{sinc}(\mathbf{f} \cdot \mathbf{v}_l t_i) \exp(2\pi i \mathbf{f} \cdot \mathbf{v}_l t_d) \times \frac{g(e^{ib_l} - a)}{1 - 2a\cos b_l + a^2} \tag{34}$$

with a and b_l defined as above.

3.2 Noise

For WFS noise the function $G_n = \mathcal{R}(\nu_n)$, and the closed-loop PSD is given by

$$\Phi_{noise}(\mathbf{f}) = \left\langle \left|\mathcal{F}\left\{s_0(\mathbf{x}, t)\right\}\right|^2 \right\rangle \tag{35}$$

$$= \left\langle \left| \mathcal{F}\left\{ g \sum_{n=1}^{\infty} (1-g)^{n-1} \mathcal{R}[\nu_n(\mathbf{x}, t)] \right\} \right|^2 \right\rangle \tag{36}$$

$$= g^2 \widetilde{\mathcal{R}}^\dagger \widetilde{\mathcal{R}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (1-g)^{m+n-2} \left\langle \widetilde{\nu}_m^\dagger(\mathbf{f}, t)\, \widetilde{\nu}_n(\mathbf{f}, t) \right\rangle. \tag{37}$$

Assuming spatially and temporally uncorrelated noise we have that $\left\langle \widetilde{\nu}_m^\dagger \widetilde{\nu}_n \right\rangle = \delta_{mn} \Phi_\nu$, the power spectrum of the input noise ν. Defining $a = (1-g)^2$ we have that

$$\Phi_{noise}(\mathbf{f}) = g^2 \widetilde{\mathcal{R}}^\dagger \widetilde{\mathcal{R}}\, \Phi_\nu(\mathbf{f}) \sum_{n=1}^{\infty} a^{(n-1)}. \tag{38}$$

The sum is a geometric series with the closed form $1/(1-a)$, and substituting the form for the reconstructor from (9) gives the final expression

$$\Phi_{noise}(\mathbf{f}) = \frac{g}{2-g} \times \frac{\Phi_\nu(\mathbf{f})}{\mathrm{sinc}^2(\mathbf{f}d)} \left(\frac{1}{f_x^2} + \frac{1}{f_y^2} \right). \tag{39}$$

3.3 Aliasing (open loop)

In this section $\varphi = \varphi_\perp$, so for aliasing $G_n = \mathcal{R}[\mathcal{M}(I_n)]$, and perfect reconstruction can no longer be invoked. This, generally speaking, leads to a mess. Doing the calculation first for the open-loop case (the closed loop adjustment will be easier to implement afterward this way), the form is as before

$$\Phi_{alias}(\mathbf{f}) = \left\langle |\mathcal{F}\{s_0(\mathbf{x},t)\}|^2 \right\rangle \;=\; \left\langle \left|\widetilde{\mathcal{R}}\widetilde{\mathcal{M}}[\widetilde{I}_n]\right|^2 \right\rangle. \tag{40}$$

Taking this one step at a time, we have that

$$\widetilde{\mathcal{M}}[\widetilde{I}_n](\mathbf{f},t) = \mathrm{comb}(\mathbf{f}d) \, * \, \left[\mathrm{sinc}(\mathbf{f}d) \times 2\pi i \mathbf{f}\, \widetilde{I}_n(\mathbf{f},t)\right]. \tag{41}$$

Introducing the shorthand notation $\mathbf{f}_m = \mathbf{f} - \mathbf{m}d^{-1}$, we can evaluate this as

$$\widetilde{\mathcal{M}}[\widetilde{I}_n](\mathbf{f},t) = 2\pi i \sum_{\mathbf{m}} \mathbf{f}_m \,\mathrm{sinc}(d\mathbf{f}_m) \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f}_m,t)\,\mathrm{sinc}(\mathbf{f}_m \cdot \mathbf{v}_l t_i)\exp(2\pi i \mathbf{f}_m \cdot \mathbf{v}_l t_d). \tag{42}$$

Including the reconstructor we can write

$$\widetilde{\mathcal{R}}\widetilde{\mathcal{M}}[\widetilde{I}_n](\mathbf{f},t) = \frac{1}{2\,\mathrm{sinc}(\mathbf{f}d)} \times \sum_{\mathbf{m}} A(\mathbf{f}_m,t) \sum_{l=1}^{N_l} \widetilde{\varphi}_l(\mathbf{f}_m,t) E_l(\mathbf{f}_m,t), \tag{43}$$

where we defined the two quantities

$$A(\mathbf{f}_m,t) \;=\; (\mathbf{f}^{-1}\cdot \mathbf{f}_m)\,\mathrm{sinc}(d\mathbf{f}_m), \tag{44}$$

$$E_l(\mathbf{f}_m,t) \;=\; \exp(2\pi i \mathbf{f}_m \cdot \mathbf{v}_l t_d)\,\mathrm{sinc}(\mathbf{f}_m \cdot \mathbf{v}_l t_i). \tag{45}$$

Evaluating the modulus squared and applying ensemble averaging gives

$$\Phi_{alias}(\mathbf{f}) = \frac{1}{4\,\mathrm{sinc}^2(\mathbf{f}d)} \times \sum_{\mathbf{m}}\sum_{\mathbf{m}'}\sum_{l}\sum_{l'} A^\dagger(\mathbf{f}_m,t)A(\mathbf{f}_{m'},t)E_l^\dagger(\mathbf{f}_m,t)E_{l'}(\mathbf{f}_{m'},t)\left\langle \widetilde{\varphi}_l^\dagger(\mathbf{f}_m,t)\widetilde{\varphi}_{l'}(\mathbf{f}_{m'},t)\right\rangle. \tag{46}$$

We are saved from total catastrophe by assuming that separate turbulence layers are uncorrelated, and that turbulence at different spatial frequencies are uncorrelated also. The last term then becomes $\langle \widetilde{\varphi}_{lm}^\dagger \widetilde{\varphi}_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}\langle|\widetilde{\varphi}_{lm}|^2\rangle$, and the E_l term loses its complex exponential to the modulus, which leaves the almost manageable final expression:

$$\Phi_{alias}(\mathbf{f}) = \frac{0.00575}{\mathrm{sinc}^2(\mathbf{f}d)} \times \sum_{\mathbf{m}\neq(0,0)} (|\mathbf{f}_m|^2 + f_0^2)^{-11/6}(\mathbf{f}^{-1}\cdot\mathbf{f}_m)^2\,\mathrm{sinc}^2(d\mathbf{f}_m)\sum_{l=1}^{N_l} r_{0l}^{-5/3}\,\mathrm{sinc}^2(\mathbf{f}_m\cdot\mathbf{v}_l t_i). \tag{47}$$

Note that the origin $\{m = 0, n = 0\}$ is exluded from the double sum, but otherwise it runs over infinity. The terms of the sum quickly tend toward zero thanks to the steep power law of Kolmogorov turbulence, so in practise no more than a handful of terms need to be summed in each direction; this makes the expression possible to compute.

3.4 Aliasing (closed loop)

Looking to the closed loop modifications, we are now rewarded for having done most of the cumbersome calculations already. The adjustment is simplest to account for by entering it into equation (45) of the E_l term by simply substituting

$$\exp(2\pi i \mathbf{f}_m\cdot\mathbf{v}_l t_d) \;\;\rightarrow\;\; g\sum_{n=1}^{\infty}(1-g)^{n-1}\exp[2\pi i \mathbf{f}_m\cdot\mathbf{v}_l(t_d + n t_i)] \tag{48}$$

$$= \;\; g\,\exp(2\pi i \mathbf{f}_m\cdot\mathbf{v}_l t_d) \times \frac{e^{ib_l} - a}{1 - 2a\cos b_l + a^2}. \tag{49}$$

We had evaluated the sum already in the expression (34) for Γ_l, and the form (22) is retained with $\mathbf{f}$ replaced by $\mathbf{f}_m$ in the definition of b_l. We now re-evaluate $E_l^\dagger E_l$. Defining $c = 2\pi \mathbf{f}_m \cdot \mathbf{v}_l t_d$ and $z = e^{ic}(e^{ib_l} - a)$ as the complex numerator of (49) we have that

$$\begin{aligned} z^\dagger z &= e^{-ic}(e^{-ib_l} - a) \times e^{ic}(e^{ib_l} - a) && (50) \\ &= 1 + a^2 - a(e^{ib_l} + e^{-ib_l}) && (51) \\ &= 1 + a^2 - 2a\cos b_l, && (52) \end{aligned}$$

which is exactly the denominator of (49). Hence the modified expression for $E_l^\dagger E_l$ in closed loop is simply

$$E_l^\dagger(\mathbf{f}_m, t) E_l(\mathbf{f}_m, t) = \frac{g^2 \operatorname{sinc}^2(\mathbf{f}_m \cdot \mathbf{v}_l t_i)}{1 - 2a\cos b_l + a^2}, \tag{53}$$

And the closed loop aliasing PSD is finally

$$\Phi_{alias}(\mathbf{f}) = \frac{0.00575}{\operatorname{sinc}^2(\mathbf{f}d)} \times \sum_{\mathbf{m}\neq(0,0)} \frac{(\mathbf{f}^{-1} \cdot \mathbf{f}_m)^2 \operatorname{sinc}^2(d\mathbf{f}_m)}{(|\mathbf{f}_m|^2 + f_0^2)^{11/6}} \sum_{l=1}^{N_l} r_{0l}^{-5/3} \frac{g^2 \operatorname{sinc}^2(\mathbf{f}_m \cdot \mathbf{v}_l t_i)}{1 - 2a\cos b_l + a^2}, \tag{54}$$

where $a = 1 - g$ and $b_l = 2\pi \mathbf{f}_m \cdot \mathbf{v}_l t_i$.

4 Sample numerical results

See Figures 1-3. Although the aliasing error is a function of the gain, the shape of the PSD is almost unchanged within realistic gain values (e.g. 0.1-0.9). The right-hand graph of figure 3 shows the temporal averaging effect, that the aliasing error decreases as the integration time increases.

5 Summary

Comparing to the results presented in [2], the major differences in the current text are

1. Closed analytical form of the loop recursion function S_l (32) derived
2. Simpler (and more accurate) closed-loop servo-lag term, by the new derivation of S_l
3. Completely new derivation of the aliasing term (hopefully correct this time), also utilizing S_l
4. Noise expression the same, but generalized for any shape input power spectrum Φ_ν

References

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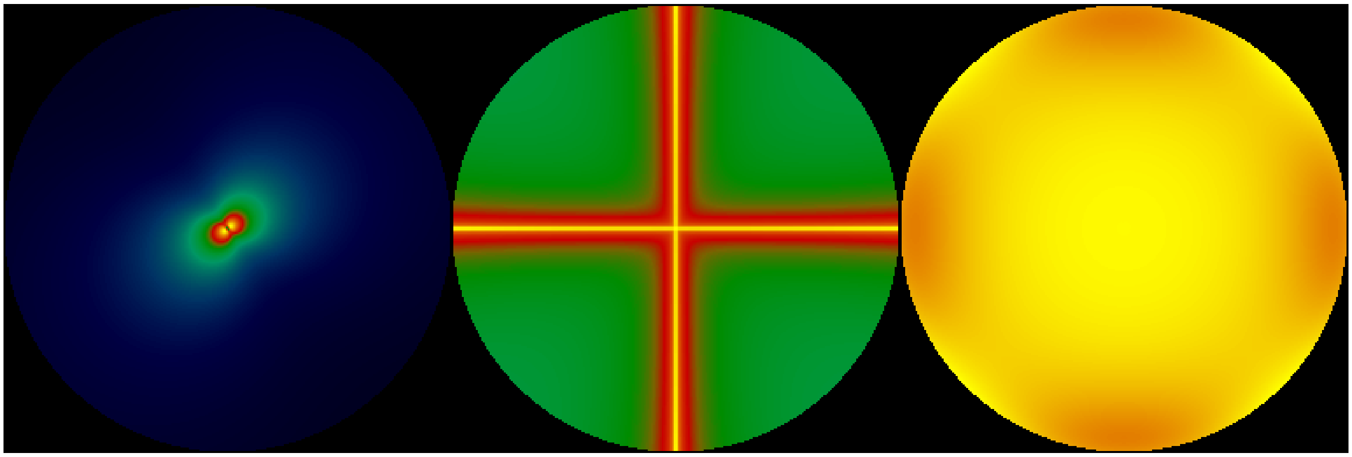

Figure 1: Servo-lag, noise and aliasing PSDs (all plotted with different stretch).

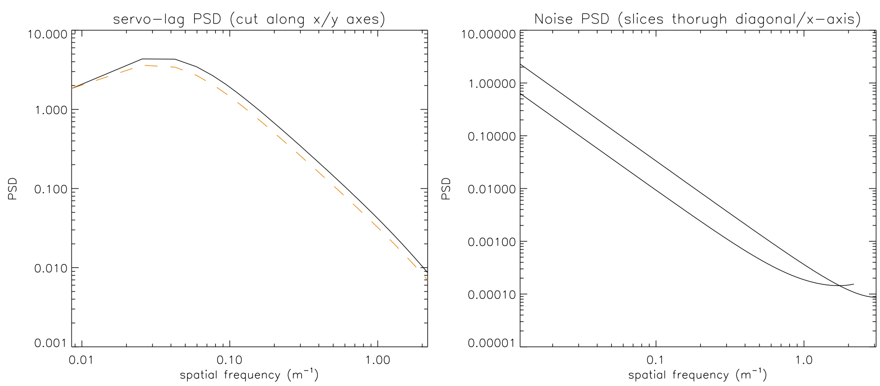

Figure 2: One-dimensional cuts through the 2D PSDs in Figure 1. Servo-lag and noise can be approximated over the majority of the frequency range by $-5/3$ and -2 power laws, respectively. Aliasing is essentially flat.

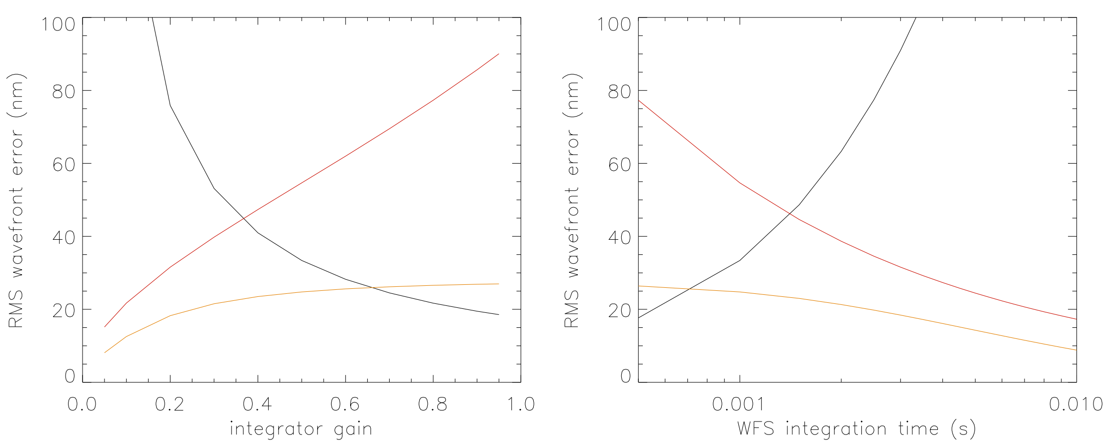

Figure 3: Servo-lag (black line), noise (red) and aliasing (orange) wavefront errors as functions of the integrator gain g (left) and the WFS integration time t_i (right).