SLGLAO

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Abstract: The properties of an adaptive optics system that utilizes a single laser beacon are analyzed. Scaling laws for field of view and quality of correction are derived using a geometrical argument. It is shown that this adaptive optics architecture permits a trade between field of view and quality of correction. More precise models of the field dependent point spread function (PSF) are derived using covariance calculations. These PSF's are then used to determine the performance of a near infrared multiobject spectrograph on a 30 meter telescope.

1. INTRODUCTION

The concept of Ground Layer AO (GLAO) was introduced by Rigaut [1] as a means of providing a partially compensated wavefront over a relatively wide field of view. The GLAO concept relies on the use of multiple laser beacons distributed over a field of order 5 to 10 arcminutes. Light from each laser is collected by the telescope and imaged through a wavefront sensor to yield an estimate of the wavefront phase in the direction of the beacon. The wavefront phase measurements from these beacons are averaged, which suppresses uncorrelated contributions that arise from higher altitude turbulence and reinforces correlated ones that arise from lower altitude turbulence. In this way, the contribution from ground layer turbulence is isolated, and may be applied to a deformable mirror conjugate to the pupil plane of the telescope. This correction is valid over a field of view much wider than that of a traditional single conjugate AO system, as the ground layer is common to all directions on the sky. However, removing the ground layer alone is not sufficient to achieve diffraction limited performance, and the PSFs delivered by this system show small reductions in width compared to seeing limited PSFs.

Recently, Tokovinin [2] suggested that a small aperture telescope could use a single Rayleigh beacon to drive a ground layer mirror. This paper argues that on a 4.2 meter telescope, such an AO architecture can reduce the PSF width by about a factor of two over a 3 arcminute field of view. One interpretation of this result is that the small footprint of the Rayleigh beacon metapupil on higher altitude layers effectively downweights the contribution from these layers, so that the ground layer contribution dominates the laser beacon wavefront phase. To distinguish the single laser concept from traditional GLAO, this paper refers to the former as single laser GLAO (SLGLAO). The success of SLGLAO for Rayleigh beacons on small aperture telescopes suggests an analysis of a SLGLAO system for larger aperture telescopes that uses a single sodium laser beacon to compensate a single deformable mirror. This is one of the simplest AO architectures, and a number of systems of this kind are currently in operation. This paper presents such an analysis.

2. FOCAL AND ANGULAR ANISOPLANATISM

A SLGLAO system suffers from the effects of both focal and angular anisoplanatism. The former effect arises because light from the laser beacon samples a conical volume of atmospheric turbulence, while that from the science target samples a cylindrical volume. In driving an adaptive mirror to compensate the laser, light from the science target is not perfectly compensated.

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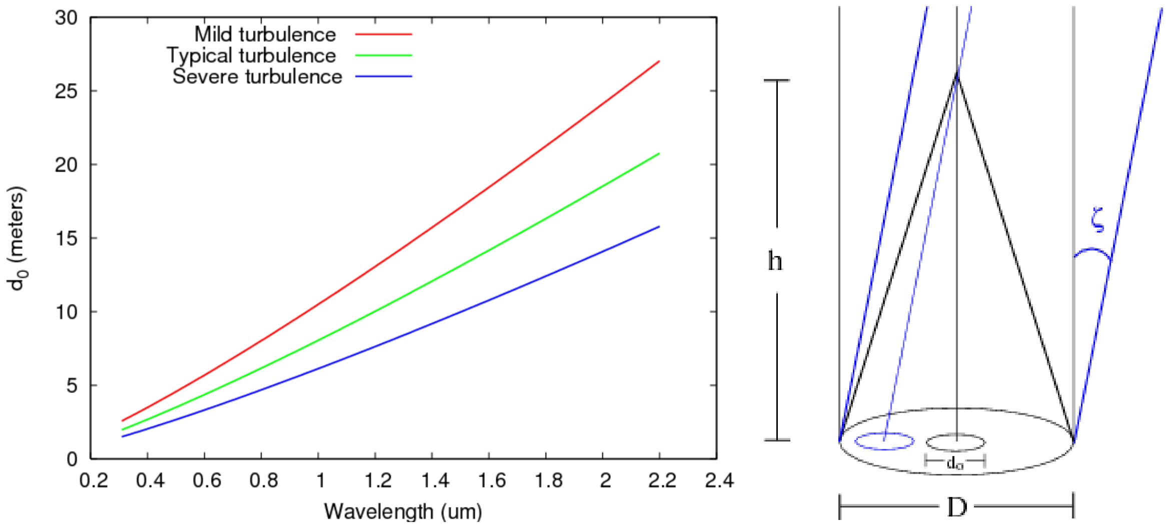

Figure 1. On the left, the focal anisoplanatism parameter d_0 is plotted vs. observing wavelength for a 90 km laser beacon and a range of astronomical turbulence conditions. On the right, the geometrical effects of combining focal and angular anisoplanatism are illustrated.

The residual error due to focal anisoplanatism [3] is $\sigma^2_{\mathrm{FA}} = (D/d_0)^{5/3}$, where D is the aperture diameter of the telescope and d_0 is given by

$$d_0 \;=\; k^{-6/5}\left[\frac{.5\,(\sec\xi)^{8/3}}{h^{5/3}}\int dz\; z^{5/3}C_n^2(z) - \frac{.452\,(\sec\xi)^3}{h^2}\int dz\; z^2 C_n^2(z)\right]^{-3/5}$$

Here k is the wavenumber, $C_n^2(z)$ is the vertical turbulence profile, ξ is the zenith angle, and h is the distance to the laser beacon. This expression is valid for a beacon that lies above all atmospheric turbulence - an assumption made throughout this paper. Note that d_0 is very nearly proportional to h. The left side of Figure 1 shows the dependence of d_0 on wavelength for a range of different astronomical turbulence conditions.

Angular anisoplanatism occurs when there is an angular offset between the guide star and the science target. In driving an adaptive mirror to compensate the guide star, light from the science target is again imperfectly corrected. When guiding on a star at infinite range, the residual error due to angular anisoplanatism is approximated by $\sigma^2_{\mathrm{AA}} = (\theta/\theta_0)^{5/3}$, where θ_0 is the isoplanatic angle [3]. When guiding on a laser beacon, the effects of focal and angular anisoplanatism interact in a rather subtle way, as shown by the following geometrical argument.

Consider the case of a laser beacon pointed at a science target directly overhead, as shown on the right side of Figure 1. There is a ray from the science target that intersects the apex of the laser cone and hits the center of the pupil. This ray traverses the same path through the atmospheric turbulence as the ray from the laser that hits the center of the pupil. When a correction is applied to an adaptive mirror to compensate the laser beacon wavefront, this point on the science wavefront is perfectly compensated. As one moves away from this point, residual errors in the science wavefront increase. The offset at which these errors exceed about a radian defines the lateral coherence scale, which is equal to d_0.

Next, consider a science target at angular offset ζ. For sufficiently small values of ζ there is a ray from the science target that intersects the apex of the laser cone and lands at a different point within the telescope pupil. As above, this point is perfectly compensated by the adaptive optics system, and there is a region of size d_0 over which the wavefront of the science target remains coherent. This argument holds for all science targets that lie at angles less than the critical angle $\zeta_c = D/2h$, beyond which the coherence patch falls outside of the pupil. This angle is given by

$$\zeta_c = 35 \text{ arcsecs} \left(\frac{D}{30 \text{ meters}}\right)\left(\frac{90 \text{ km}}{h}\right)$$

This geometrical argument permits a qualitative description of the SLGLAO PSF. Since the science wavefront contains a coherence patch of size d_0, the PSF will have a core of width λ/d_0 that contains a fraction $(d_0/D)^2$ of the total energy. This PSF will be roughly constant over a field of view of size $2\zeta_c$.

These scaling laws illustrate that the SLGLAO architecture permits a trade between field of view and quality of correction. As argued above, d_0 is nearly proportional to h. As the height of the beacon increases, the width of the core decreases and the fraction of energy lying within this core increases. However, the field of view decreases. Conversely, as the telescope diameter increases the fraction of light within the core decreases and the field of view increases.

3. COVARIANCE PSFS

The scaling laws discussed in the previous section that govern the behavior of a SLGLAO system are not sufficient to derive concrete performance estimates for this architecture. One problem lies in the fact that at near infrared wavelengths d_0 is comparable to the size of the telescope apertures of interest, leading to a breakdown in the scaling laws. To make further progress, one must formulate a more precise model of the PSF that captures the effects of focal and angular anisoplanatism.

It is possible to formulate such a model through a covariance analysis. In this type of analysis, one computes the phase covariance $< \phi_a(\vec{\mathbf{r}}_1)\phi_b(\vec{\mathbf{r}}_2) >$ between two different sources a and b at two different locations $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ in the pupil plane. Semianalytic expressions for this quantity have been computed by Tyler [4] for sources at arbitrary range and at finite angular offset, assuming that atmospheric turbulence follows Komolgorov statistics. These expressions have been incorporated into Arroyo: a publically available software library for the simulation of adaptive optics systems [5]. Using this functionality, one may compute the residual wavefront phase after compensation by the adaptive optics system. The structure function of the residual phase and the associated optical transfer function may then be computed using standard techniques [6]. Fourier transformation of the optical transfer function yields the PSF. This model incorporates the effects of both focal and angular anisoplanatism through the expression for the covariance, which depends on the range and angular offset of the two sources. The model also incorporates the turbulence profile and aperture diameter through the covariance expression.

Figure 2 shows radial cuts through the 1.65 μm PSFs delivered by a SLGLAO system employing a 90 km guide star for apertures between 5 and 30 meters under typical astronomical turbulence conditions. The errors due to focal anisoplanatism increase with D, causing the on-axis SLGLAO Strehl ratio to fall. However, the Strehl ratio falls more slowly with angular offset from the laser beacon as D increases. This is most apparent at an angular offset of 20 arcseconds. This offset is less than ζ_c for a 30 meter aperture, but greater than those for 5 and 10 meter apertures. Another important feature illustrated by these curves is that the marginal benefit of a SLGLAO system over the seeing limit increases with D. For a 30 meter aperture, SLGLAO improves the Strehl ratio by about 2 orders of magnitude over a 1 arcminute field.

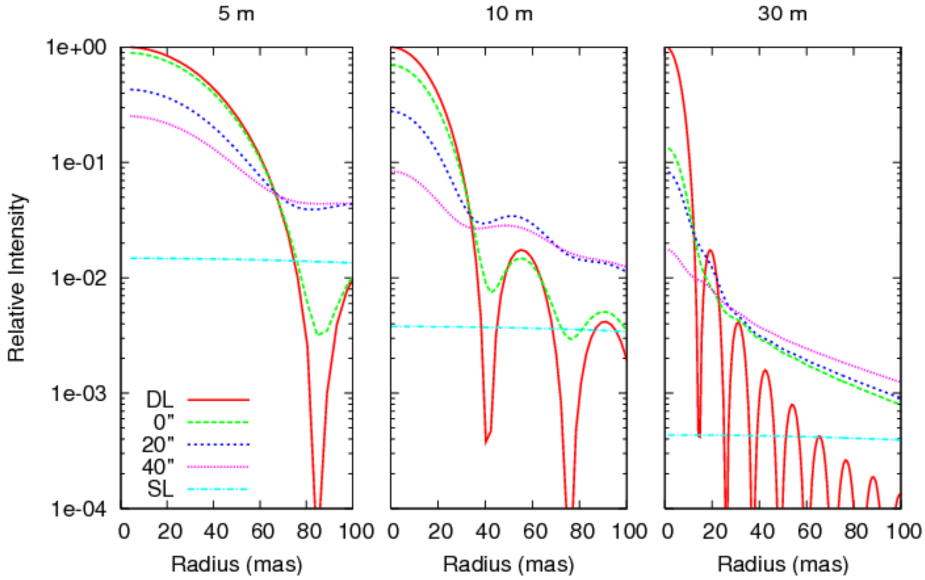

Figure 2. Radial cuts through the PSF for diffraction limited, seeing limited and SLGLAO architectures on 5 to 30 meter apertures. SLGLAO results are shown for angular offsets of 0", 20", and 40" from the laser beacon. A guide star altitude of 90 km has been used in the computation of the SLGLAO PSFs, and a typical astronomical turbulence profile was used. These results assume perfect tip tilt compensation.

4. AO-FED NEAR IR SPECTROSCOPY

To illustrate the utility of a SLGLAO architecture, consider the application of AO-fed near IR spectroscopy on a 30 meter telescope using a 90 km sodium beacon. Such a system could feed an instrument with slits, fibers, or IFU's deployed over a field of order 1 arcminute. The performance of such a system may be quantified through the SNR of an observation. Assuming observations are background limited, one finds that $SNR \propto S\sqrt{t}/w$, where S is the slit coupling fraction, t is the integration time, and w is the slit width. With this expression, one may compare the relative performance of two architectures using the ratio of integration times required to reach the same SNR.

$$\frac{t_1}{t_2} = \left(\frac{S_2}{S_1}\frac{w_1}{w_2}\right)^2$$

The slit coupling fraction S may be computed as a function of slit width using the PSF's from the previous section, and an optimal slit width may be chosen to maximize the SNR. When the fraction of energy in the core is sufficiently large, the optimal slit width is of order λ/d_0. For typical values of d_0, the optimal width is of order 20 mas at 1.65 μm and has a weak wavelength dependence of $\lambda^{-1/5}$. The fraction of energy in the core decreases with decreasing wavelength, and at a critical wavelength the optimal slit width jumps to a value comparable to that of the seeing limited halo. Figure 3 shows the integration time ratio for SLGLAO and seeing limited observations as a function of observing wavelength. The transition to a 20 mas slit occurs where the 0" and 60" SLGLAO curves level off between .8 and 1 μm. These results illustrate the significant performance improvements delivered by a SLGLAO system throughout the near infrared over fields of order 2 arcminutes.

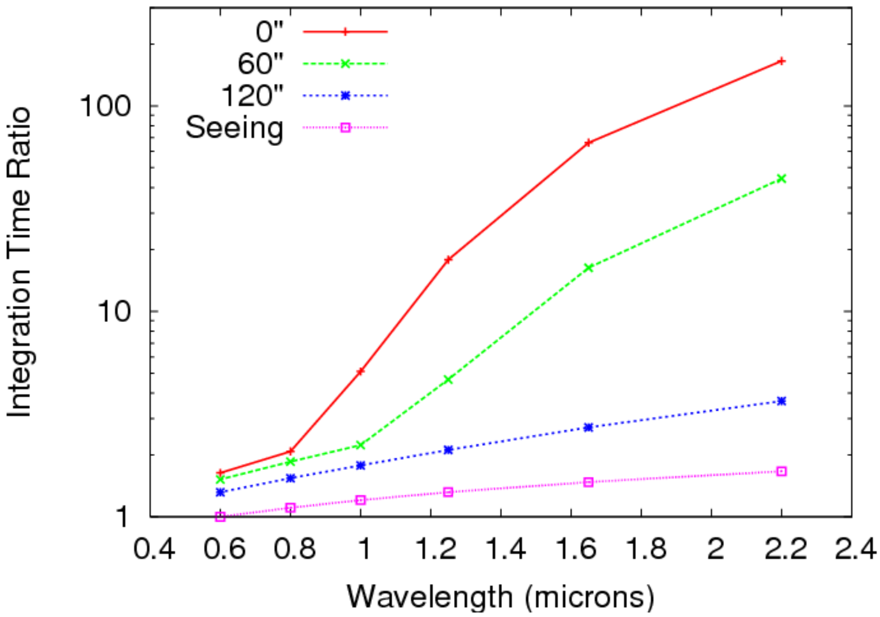

Figure 3. Integration time ratio vs. observing wavelength for a SLGLAO system employing a 90 km laser beacon. Results are shown for angular offsets of 0", 60" and 120" from the laser beacon. The integration time ratio has been computed relative to the seeing limited result at .6 μm.

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