# Stellar scintillations as a remote atmospheric wave-front sensor

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Stellar scintillations are considered noise in adaptive-optics sensors and are measured for calibration purposes only. We propose to use scintillations to provide direct instantaneous information about the structure of the atmosphere. As a result it will be possible to increase the field of view provided by adaptive optics. The scintillation pattern is created when stellar light is diffracted by high-altitude turbulence. Alternatively, this pattern can be viewed as a Laplacian of this turbulence and can thus be inverted to estimate it. The measurement is limited by the intensity and the angular size of the reference star, by the height distribution of the atmospheric turbulence, and by the detector resolution and spectral response. 

Adaptive-optics systems can correct only a narrow field of view. This is because they measure the atmosphere by integration along a cylinder or a cone extending from the telescope aperture to the star (natural or artificial). A number of deformable mirrors could compensate the phase over an extended field. To accomplish this, one can probe a larger atmospheric volume, using a multitude of guide stars.[1–3] The correction will best succeed if each deformable element can be conjugated to an isolated turbulent layer, as suggested by atmospheric measurements.[3–5] Turbulence can then be represented by a set of thin phase screens[3] if the thickness and the altitude of the layers are such that diffraction effects within the layers are negligible.[2]

Recently, Angel[6] proposed the use of adaptive optics to locate planets next to bright, nearby stars. The dynamic range of the scheme is restricted by scintillation, which is produced by high-altitude turbulence.[7] Since candidates for such a program are bright point sources, their scintillation can also serve to measure the high turbulence.[8] We examine this option for adaptive optics in general, as a supplement to the standard methods.

A plane wave, originating from a point source at infinity, undergoes minute changes as it traverses various thin atmospheric layers. Using either arguments of conservation of energy and geometrical optics or Fermat's principle,[9] we get a direct relationship between scintillation and refractive-index variations. Let the refractive index of air be $n(\mathbf{r}, z) = 1 + \mu(\mathbf{r}, z)$, where $\mu(\mathbf{r}, z) \ll 1$ and $\mathbf{r} = (x, y)$. The logarithm of the intensity pattern at ground level ($z = 0$) is [7,9]

$$\chi(\mathbf{r}) \triangleq \ln \frac{I(\mathbf{r})}{\bar{I}} = -\int_0^\infty z\nabla^2\mu(\mathbf{r}, z)\mathrm{d}z \approx -\sum_{i=1}^{L} z_i\nabla^2\mu_i(\mathbf{r}), \tag{1}$$

where $\nabla = \partial/\partial x + \partial/\partial y$, $\bar{I}$ is a constant average intensity, and we have assumed that the refractive-index perturbations ($\mu \neq 0$) occur only inside $L$ layers. Now it is rather obvious why high-altitude turbulence causes scintillation: because of altitude weighing there is a lever effect, and turbulence at 100 m will have to be 80 times stronger to have the same effect as turbulence at 8 km. This is usually not the case, and we can therefore disregard low-altitude atmospheric aberrations.

The intensity and the phase of the propagating wave front can also be related by the irradiance transport equation,[10,11] derived from Fresnel diffraction:

$$\partial I(\mathbf{r}, z)/\partial z = -\nabla I(\mathbf{r}, z) \cdot \nabla\phi(\mathbf{r}, z) - I(\mathbf{r}, z)\nabla^2\phi(\mathbf{r}, z)\,. \tag{2}$$

Roddier was able to relate this equation to curvature sensing, as implemented by him.[12] In our case the intensity arriving from the source is constant, so $\nabla I = 0$, and we get an equivalent to relation (1).

Now we turn to a third viewpoint: direct Fresnel diffraction. For simplicity, let us assume that the atmosphere is composed of two main layers, relatively thin, one high above the telescope and the other next to it. Phase errors are added to the cascading beams only inside these layers, while the field undergoes Fresnel diffraction between them. The Fresnel approximation is valid above elevation $H \gg k^{1/3}\rho^{4/3}/2$, where $k = 2\pi/\lambda$ and $\rho$ is the lateral (horizontal) distance between the scattering point and the measurement point. This condition is fulfilled[5] for $\rho = 1$ m, $\lambda = 0.5$ $\mu$m, and $H \gg 116$ m. $H$ can be even smaller because of the principle of stationary phase.[13]

We describe a field of amplitude $F$ arriving from a star as $O(\mathbf{r}) = F$. Passing through the top layer, it accumulates a phase angle $\phi(\mathbf{r})$; it is this phase angle that we wish to estimate. Once below the layer, say, at altitude $h$, the field can be described as $P(\mathbf{r}) = O(\mathbf{r})\exp i\phi(\mathbf{r}) = F \exp i\phi(\mathbf{r})$. Then there is free-space propagation until the top of the boundary

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layer. Here we write the field as a convolution of the former field and a Fresnel point-spread function[13]:

$$Q(\mathbf{r}) = P(\mathbf{r}) * (2\lambda h)^{-1} \exp[ikh(1 - r^2/2h^2)]. \quad (3)$$

As it traverses the turbulent boundary layer, the field $Q(\mathbf{r})$ accumulates an additional phase $\psi(\mathbf{r})$. Thus the field at the telescope aperture is $R(\mathbf{r}) = Q(\mathbf{r})\exp i\psi(\mathbf{r})$. The intensity of this field is $S(\mathbf{r}) = |R(\mathbf{r})|^2 = |Q(\mathbf{r})\exp i\psi(\mathbf{r})|^2 = |Q(\mathbf{r})|^2$, *independent* of the effects of the boundary layer. However, the phase of the field can be written as the sum of the phases of the two layers,[5,7] $\arg\{R(\mathbf{r})\} = \phi(\mathbf{r}) + \psi(\mathbf{r})$. This is the phase that one usually measures with a wave-front sensor.

Can we benefit from this model? Instead of calibrating scintillation "noise" away, we use it as additional information about the atmosphere. Assume that the measured field intensity $\hat{S}(\mathbf{r})$ is sampled densely enough but that it is contaminated by additive detection noise. Let us take its logarithm, $\hat{\chi}(\mathbf{r}) = \ln[\hat{S}(\mathbf{r})/\overline{S}]$, where $\overline{S}$ is the average of the intensity over the aperture and over many realizations. Inverting the Fresnel transform [Eq. (3)] is difficult if we have only the intensities $S(\mathbf{r})$. Fortunately, solving Poisson's equation [relation (1)] is simple and robust. Essentially, we have to integrate twice the logarithm of the intensity fluctuations $\hat{\chi}(\mathbf{r})$ over the telescope aperture. Note the similarity to curvature sensing,[12] albeit with worse boundary conditions. The lowest modes are lost—piston errors are inconsequential; tip and tilt of the top layer can be combined with attitude correction for all the layers. Higher modes whose Laplacian is zero cannot be retrieved without boundary conditions,[14,15] which are partially replaced here by knowledge of the spatiotemporal spectrum of the turbulence[16] and of the average intensity and phase on the outer scale. The solution to Poisson's equation is known to be well posed and not sensitive to variations in $\hat{h}$, the estimated altitude of the turbulence.[4]

Taking the Laplacian of the wave front [relation (1)] is equivalent to multiplication of it by a quadratic filter in the Fourier domain, $C(w) = -1/4\pi^2 w^2$, where $w^2 = u^2 + v^2$ is the Fourier frequency. Hence the double integration amounts to deconvolution by the same quadratic
numbers, one can apply a Wiener filter, $W(w) = C^{-1}(w)F_A(w)/[C^{-2}(w)F_A(w) + F_N(w)]$, where $F_A(w)$ and $F_N(w)$ are the power spectra of the atmospheric wave fronts (at the top layer) and of the detection noise present in the log intensity $\hat{\chi}(\mathbf{r})$. All these functions are real and centrosymmetric. They can be either measured and updated during observation or constructed from accepted models: the power spectrum of a thin layer is[5,7]

$$F_A(w) = 6.9 \times 2^{2/3} \sin(5\pi/6)\Gamma^2(11/6) \times \pi^{-2} r_0^{-5/3} w^{-11/3} \triangleq \gamma w^{-11/3}, \quad (4)$$

where we have neglected inner- and outer-scale effects and $r_0$ is Fried's parameter for the high turbulence.[5] Thus $\gamma = 0.0672k^2 \int_h^\infty C_n^2(z)\mathrm{d}z$. The power spectrum of the logarithm of the intensity is limited by shot noise, $F_{\mathrm{ls}}(\nu) \approx (1.022\nu - 0.967 \log \nu + 8.10\nu \exp -\nu)^{-1}$, where $\nu$ is the number of photons corresponding to $\bar{S}$. The approximation is good to 1% down to $\nu = 1$ photon and slightly worse for weaker fluxes. If the scintillation is mild, with variance of the intensity $\sigma^2 < 0.04$, then, within 1% again, $F_N \approx (1 + 1.2\sigma^2)F_{\mathrm{ls}}$. Note that the intensity variance can be modeled by[7] $\sigma^2 \approx 0.077 C_n^2 k^{7/6} \hat{h}^{11/6}$. Taking only the leading term, we have $F_N(\omega) \approx 1/\overline{S}$. Hence we ignore the scintillation at each pixel and its covariance with neighboring pixels. Finally, we ignore effects that are due to the finite telescope aperture. Thus the Wiener filter will be

$$W(w) = -4\pi\overline{S}\gamma/(16\pi^2\gamma\overline{S}w^2 + w^{5/3}). \quad (5)$$

Utilization of scintillation is limited by several factors:

(a) Intensity. For $\nu$ photons per pixel the shot-noise level is $\nu^{1/2}$. It will be difficult to detect scintillation weaker than this level: intensity fluctuations equivalent to $\nu \pm \nu^{1/2}$ will yield $\hat{\chi} \approx \chi \pm \nu^{-1/2}$. This sets the lower bound of retrieved phase perturbations. However, detection noise is uncorrelated, whereas turbulence is fractal and more predictable.[16]

(b) Resolution. Because of the feeble photon flux, the pixels in wave-front sensors tend to be larger then $r_0$, whereas scintillation is much finer.[7] (The same applies for temporal integration.) In this case the phase should be solved for by optimization, under the constraints of atmosphere and noise.

(c) Color. The Fresnel approximation is explicitly wavelength dependent [Eq. (3)], and so is the atmospheric power spectrum [Eq. (4)]. Experimental evidence is rather scant,[17] so we tried a simple laboratory experiment. Using a sodium lamp imaged onto a small pinhole, we passed a well-collimated beam through a boundary layer of air over a hot oven. No color effects were discerned at Fresnel distances, even under strong scintillation. If scintillation varies with color, such as for observation far from zenith, then the bandwidth must be limited. However, sodium-layer laser guide stars possess a narrow band and might produce scintillation when small. One can test the validity of the method and of the color effects can be tested by measuring scintillation in two bands simultaneously. With one band used to retrieve the phases, it should be possible to predict the pattern in the second band.

(d) Source size. Planets scintillate less then stars: our eyes average wave fronts cascading through the atmosphere at different source directions. The angular size of objects prone to scintillation is[7] $\gamma_0 = \sqrt{2\lambda/\pi h}$. For $h \approx 8$ km and $\lambda = 0.589$ $\mu$m, this means objects smaller than $\gamma_0 = 1.4$ arcsec. Bright asteroids and small sodium beacons are near this limit.

(e) Field of view. The corrigible wave front is limited to the measured volume between the source and the telescope aperture. However, a second deformable mirror conjugate to the top layer widens the total field of view.

(f) Isolation of layers. Scintillation provides information about the top layer, and conventional wave-front sensing provides information about the sum of

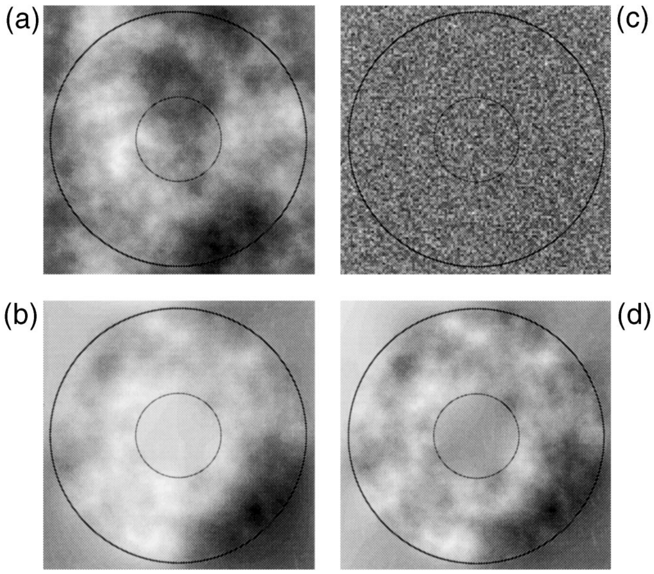


Fig. 1. (a) Single realization of high-altitude wave fronts, $r_0 = 10$ cm. $128^2$ array, each element 2.5 cm. The telescope aperture (3-m diameter, 1/3 obscuration) is shown only for reference. (b) Wideband scintillation pattern at the telescope aperture resulting from the image in (a); the average number of photons per pixel is 100, as might be detected from a magnitude 6 star (0.5 total efficiency, 3-ms integration, $\lambda = 550$ nm, $\Delta\lambda = 300$ nm). (c) Direct deconvolution of the log-intensity image in (b): reconstructed wave fronts using scintillation inside the aperture. (d) Wiener deconvolution: reconstruction using models of the atmosphere and the noise. The rms difference between the original and the calculated wave front over the aperture, with the global slope removed, was 0.77 wave for both deconvolutions. At 1000 photons per pixel (a magnitude 3.6 star) both results were indistinguishable from the original.

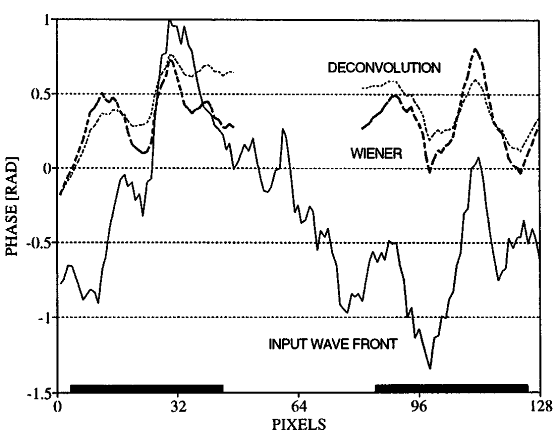


Fig. 2. Cuts across the centers of the input and resultant wave fronts using direct and Wiener deconvolutions (Fig. 1). The telescope is marked as two bars at the bottom. Low and high frequencies are lost.

the two layers. A simultaneous solution for both will reduce their covariance.

(g) Thicker layers. A thick top layer degrades the isoplanatic angle.[3] Relation (1) or (2) can be solved iteratively for the separate contributions of sublayers.

To assess the method, we ran many blind tests: we simulated scintillation, using Fresnel propagation, and tried to recover the original wave fronts using inversion of the Laplacian. We created fractal wave fronts[16] and convolved them with the Fresnel kernel [Eq. (3)], simulating the propagation. The resultant field was squared to yield the intensity at the aperture plane. The patterns were found to be independent of wavelength. Poisson noise was applied to the result, and only the pixels inside the aperture were taken. This completed the simulation of scintillation (Fig. 1).

To recover the original wave front, we applied two different deconvolutions to the normalized logarithm of the intensity. The first was a simple inversion of the Laplacian. Then we tried a Wiener filter [Eq. (5)] that included the atmospheric spectrum and Poisson noise. Figure 1 shows the results for a low flux. At high flux the original wave front was reconstructed accurately even under low turbulence. Weaker signal resulted in loss of the lowest and highest frequencies (Fig. 2). A second-order effect was due to the lack of good boundary conditions when the diameter spanned fewer than 32 pixels.

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## References

1. J. M. Beckers, in *Proceedings of European Southern Observatory,* M.-H. Ulrich ed. (European Southern Observatory, Garching, Germany, 1988), Vol. 30, p. 693.
2. J. Shamir, D. G. Crowe, and J. W. Beletic, Appl. Opt. **32,** 4618 (1993).
3. Y. Baharav, E. N. Ribak, and J. Shamir, Opt. Lett. **19,** 222 (1994), and references therein.
4. J. Vernin and F. Roddier, J. Opt. Soc. Am. **63,** 270 (1973); J. L. Caccia, M. Azouit, and J. Vernin, Appl. Opt. **26,** 1228 (1987); **27,** 2229 (1988); F. Roddier, L. Cowie, J. E. Grves, A. Songaila, D. McKenna, J. Vernin, M. Azouit, J. L. Caccia, E. Limburg, C. Roddier, D. Salmon, S. Beland, D. Cowley, and S. Hill, Proc. SPIE **1236,** 485 (1990).
5. C. E. Coulman, Ann. Rev. Astron. Astrophys. **23,** 19 (1985).
6. J. R. P. Angel, Nature (London) **368,** 203 (1994).
7. V. I. Tatarskii, *The Effects of the Turbulent Atmosphere on Wave Propagation* (Israel Program for Scientific Translations, Jerusalem, 1971), Chaps. 1 and 3.
8. E. N. Ribak, Proc. SPIE **2426,** 128 (1994).
9. L. A. Chernov, *Wave Propagation in a Random Medium* (McGraw-Hill, New York, 1960), Chap. 2.
10. M. R. Teague, J. Opt. Soc. Am. **73,** 1434 (1983).
11. N. Streibl, Opt. Commun. **49,** 6 (1984).
12. F. Roddier, Appl. Opt. **27,** 1223 (1988); **29,** 1402 (1990).
13. J. W. Goodman, *Introduction to Fourier Optics* (Wiley, New York, 1968), Chap. 4.
14. C. Schwartz, E. N. Ribak, and S. G. Lipson, J. Opt. Soc. Am. A **11,** 895 (1994).
15. T. E. Gureyev, A. Roberts, and K. A. Nugent, J. Opt. Soc. Am. A **12,** 1932 (1995).
16. C. Schwartz, G. Baum, and E. N. Ribak, J. Opt. Soc. Am. A **11,** 444 (1994).
17. M. A. Ellison and H. Seddon, Mon. Not. R. Astron. Soc. **112,** 73 (1952).