MGSU data analysis strategy

Abstract
This text gives a brief summary of the main points of the experiment, and an overview of the different analysis approaches that may be applied to the collected data.

Introduction
The overall goal of the multiple guide star unit (MGSU) data analysis is to assess the accuracy with which tomographic turbulence estimation can be performed, using multiple natural guide stars (NGS) and Shack-Hartmann wavefront sensors (WFS). This issue is of critical importance to the design of several current and future adaptive optics (AO) instruments that will employ multiple NGS or laser guide stars (LGS) for wavefront sensing and volume turbulence estimation. It is however very hard to derive analytical scaling laws for the tomographic estimation error, and usually complex numerical simulations are required for any given system architecture. The MGSU data analysis will go some ways toward understanding the real world performance and limitations of Shack-Hartmann-based tomographic wavefront estimation.

The detailed architecture of the experiment is described elsewhere [cite]. The general scope of the experiment is to use the information from three NGS to estimate the wavefront at a fourth position, where also a fourth NGS is available to provide a “truth” measurement against which the 3-NGS prediction is compared and performance assessed. An ideal asterism for this type of estimation and validation would be an equilateral triangle (tomography sensors) with a fourth star in the center (truth sensor). Ideally one would put the HOWFS (high-order WFS) of the Palomar AO system on the central star to act as truth sensor, and train the three MGSU cameras on the encircling stars to act as the tomography system. A variation on this experiment may be carried out using only three NGS, where two are employed to make a prediction for the third, which may be located “off-axis” rather than in the central region. This experiment mimics the situation in certain MOAO (multi-object adaptive optics) designs or the off-axis sharpening of a tip-tilt NGS based on on-axis LGS tomography. The main experiment is an all-open-loop capture, which lends itself to the most general analysis approach (see next section), but captures with partial (tip+tilt) and full AO compensation in closed loop will also be done for completeness of the data set (and in case the open loop turbulence produces too strong nonlinearities).

Analysis and estimation techniques
Given simultaneous measurements from four WFSs on a 4-star asterism as described above, one can apply a number of different analysis methods that will answer different questions about the data set and the outcome of the experiment. The analysis methods that we will be considering for the MGSU data analysis include:

1. Wavefront reconstruction from centroids,
2. Statistical Waller-type estimator on centroids,
3. Zernike mode reconstruction and spatial correlation analysis,
4. Cross-validation of tomography estimation results by simulation

The first method would, in the presence of high quality data, provide the most convincing tomography validation case, while the second method would provide a relative performance measure as computed in centroid data domain. The third analysis is not so much a tomography validation as an additional experiment in atmospheric turbulence analysis that can be done with the data, and the results compared with analytical models for Zernike mode correlations in Kolmogorov turbulence. The fourth step is an additional simulation exercise, using MASS/DIMM data obtained concurrently with the MGSU data acquisition, that may be of interest mainly if we successfully complete the analysis in step 1, and to a lesser degree if only step 2 is completed.

1. Wavefront reconstruction
   This method estimates wavefronts \( w_1 \) and \( w_2 \) from centroids \( u \) and \( v \) computed from the WFS camera frames according to different linear mappings:
   \[
   w_1 = E_1 u, \tag{1}
   \]
   \[
   w_2 = E_2 v, \tag{2}
   \]

   where \( E_1 \) and \( E_2 \) are the reconstruction matrices (or the “estimators”) for the 1-NGS truth WFS system and the 3-NGS tomography system respectively. Hence \( w_2 \) will be a prediction based on a tomographic estimation from multiple NGS, and \( w_1 \) is the reference wavefront. The validity of the results from this type of analysis is critically dependant upon how well we are able to model the two systems (in particular the MGSU system) in order to compute reconstruction matrices that correspond to the real system. Critical points of modeling include WFS sub-aperture geometry and pupil registration, vignetting and partially illuminated sub-apertures, field distortion, deviations from telecentricity, non-common path aberrations, and noise. Uncertainties in these parameters may lead to significant estimation errors, though these may be mitigated to some degree by performing the wavefront reconstruction in a modal basis such as e.g. Zernike modes.

   Provided that the information required for the modeling of \( E_1 \) and \( E_2 \) can be obtained, the analysis is reduced to crunching through the synchronized frames of HOWFS and MGSU camera data, computing predictions and reference wavefronts according to equations (1) and (2) for each frame. One type of tomographic estimator that will be employed in this scenario is an open loop maximum a posteriori (MAP) or a minimum variance estimator (MVE), where the latter has the form:
   \[
   E_2 = (H_c^T W H_c + \lambda I)^{-1} H_c^T W H_x (G_x C_n^{-1} G_x + C^T C)^{-1} G_x^T C_n^{-1} \tag{3}
   \]

   This estimator is investigated in detail in e.g. Ellerbroek 2002.
2. Statistical centroid estimation
This analysis is carried out directly on the centroid data without reconstructing the wavefronts. It has the advantage that it is insensitive to the quality of the data and calibrations, as it relies only on having a sufficient number of camera frames from which to compute the statistics of a section of data. This estimator, sometimes referred to as a Wallner type estimator in the context of AO, is derived by minimizing a statistical least-squares error function

\[
\mathcal{M} = \langle \| u - Ev \|^2 \rangle \\
E^* = \arg \min_E \mathcal{M}
\]

which gives the result

\[
E^* = C_{uv}C_{vv}^{-1}
\]

where the covariance matrices are defined by

\[
C_{uv} = \langle uv^T \rangle
\]

The analysis in this case consists of two runs through the data set, the first to compute the two covariance matrices, and the second to compute the frame-by-frame prediction \( Ev \) and its RMS distance to the truth sensor measurements \( u \).

3. Zernike mode correlation analysis
This analysis would reconstruct the wavefronts from each NGS separately directly into a Zernike modal basis. By computing the correlation between mode coefficients in the different beams, a sparsely sampled structure function may be computed, which can be compared to theoretical results derived from Kolmogorov theory. The deviations from the theoretical Kolmogorov spectrum observed in this analysis could then provide information about parameters relating to the outer scale of the turbulence, its power spectrum, and spatial aliasing in the WFS.

4. Simulation and cross-validation
We may also try to reverse engineer the whole experiment by running AO simulations that model the MGSU+HOWFS system, and takes as input turbulence profile data recorded by the MASS/DIMM unit at Palomar concurrently with the MGSU data acquisition. This would also, just like the Zernike modal correlation analysis, be an experiment that yields information on the physical and modeling processes by the discrepancies observed rather than the agreement between simulated and real data.

Analysis implementation
The sequence of analysis steps, including simulation and matrix generation, will be distributed over various already existing codes plus new code written in IDL. The components include:

- IDL - new code
The main analysis, including crunching through the MGSU and HOWFS data frame by frame computing centroids, wavefront estimates, performance and correlation measures, is done in IDL, mainly by virtue of its FITS-friendly environment. For the wavefront reconstruction analysis (method 1) and simulation step, various existing AO simulation codes may be used independently of each other to generate different types of estimators. The most obvious choices are the YAO (Yorick Adaptive Optics), Arroyo, and LAOS (Linear Adaptive Optics Simulation) simulation packages. Below are show a few example pictures from a YAO simulation of the MGSU and HOWFS systems, together with a sample result screen from the main IDL analysis code running the wavefront reconstruction analysis on simulated data.

Figure 1. Simulated MGSU detector images, with three MGSU cameras pointing to three stars located on a circle of 30 arc seconds radius (YAO simulation).
Figure 2. Simulated open loop turbulence wavefront (right) and Shack-Hartmann spots (left) on the 64x64 pixel HOWFS camera (YAO simulation).

Figure 3. Sample result screen from main IDL analysis code. Top row - wavefronts: pure open loop turbulence (left); HOWFS reconstructed wavefront (center), and MGSU reconstructed wavefront (right). Bottom row - residual wavefront errors w.r.t. the true wavefront: optimal fit of 17x17 actuator DM (left); HOWFS reconstructed wavefront error (center); MGSU reconstructed wavefront error (right).