Fracture analysis for ZTF's Fused Silica Window

Shawn Callahan
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Revisit analysis with a minimum lifetime of 20 years.

The current window design is shown in drawing 1100-ZTF3401 Cryostat Window.

FEA results of ZTF's fused silica window predict a maximum stress in the window at the center. If the window is supported with a constant pressure .68" insided the outer perimeter the maximum principal stress is 8.9 MPa. If window O-ring support is removed at the corners the stress can be reduced as low as 7.4 MPa.

This analysis looks at the (slow) crack diffusion rate around these stress levels to calculate the expected lifetime as well as the probability of (fast) catastrophic failure.

This work is based on the principles of fracture mechanics using material properties measured by several sources listed below.

Conclusion: Moving the O-ring improves the factor of saftey from 4 to 4.8 with a 99% survival probability.

See references:

1) "Design strength of optical glass. " by Doyle and Kahan
http://www.sigmadyne.com/sigweb/downloads/SPIE-5176-3.pdf

2) Characteristic strength, Weibull modulus, and failure probablility of fused silica glass. by Claude Klein
See:http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/quartz_weibull.pdf

3) Glass: Science and technology by Uhlman and Kreidl
http://books.google.com/books?id=eGfANTQwut4C&pg=PA43&lpg=PA43&dq=V-K+curves+fused+silica&source=bl&ots=2Y_aLGawZ0&sig=bIKg3DWLRQDf_kF3Qt2EGZyikJA&hl=en&sa=X&ei=Sk3sUr-IEseIogSehYHAAg&ved=0CDUQ6AEwAQ#v=onepage&q=V-K%20curves%20fused%20silica&f=false

4) Stress corrosion and Static Fatigue of Glass by Weiderhorn and Bolz
http://www.ceramics.org/wp-content/uploads/2009/03/wiederhorn_stress_corrosion.pdf
5) Quartz window pressure safety by D. Shuman
http://www-eng.lbl.gov/~shuman/NEXT/pv_tdr2/quartz-window.pdf

Stress Intensity, K_i

Stress intensity, K_i, is defined as: $K_i(\sigma, a) := 2 \cdot \sigma \cdot \sqrt{a}$

stress, σ, is defined as the maximum principal stress (tensile) and crack length, a.

The critical stress intensity, K_{IC} for rupture for various brittle materials can be determined by breaking a lot of glass samples and gathering the failure statistics.

Critical Stress Intensity, K_{IC}

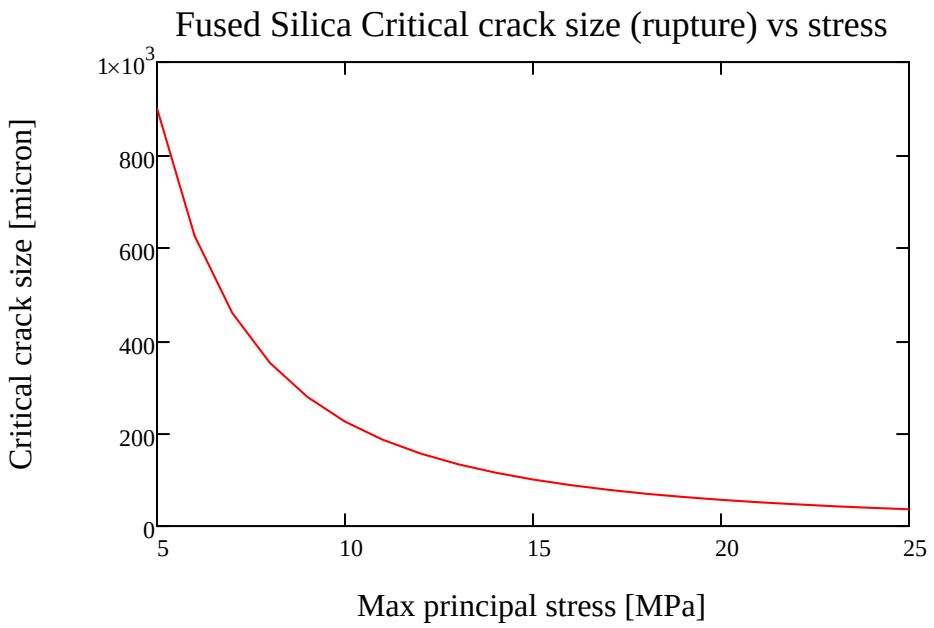
$K_{IC} := 0.3 \cdot MPa \cdot m^{.5}$ Critical crack intensity at rupture for Fused Silica[1] is a material property.

Critical crack length as a function of applied stress, a_0

$a(\sigma) := \left(\frac{K_{IC}}{2 \cdot \sigma}\right)^2$ Critical crack length for any given stress.

The following graph demonstrates critical crack length vs.stress.

$i := 5 .. 25$



Crack velocity vs. stress intensity, V-K curves for various glasses

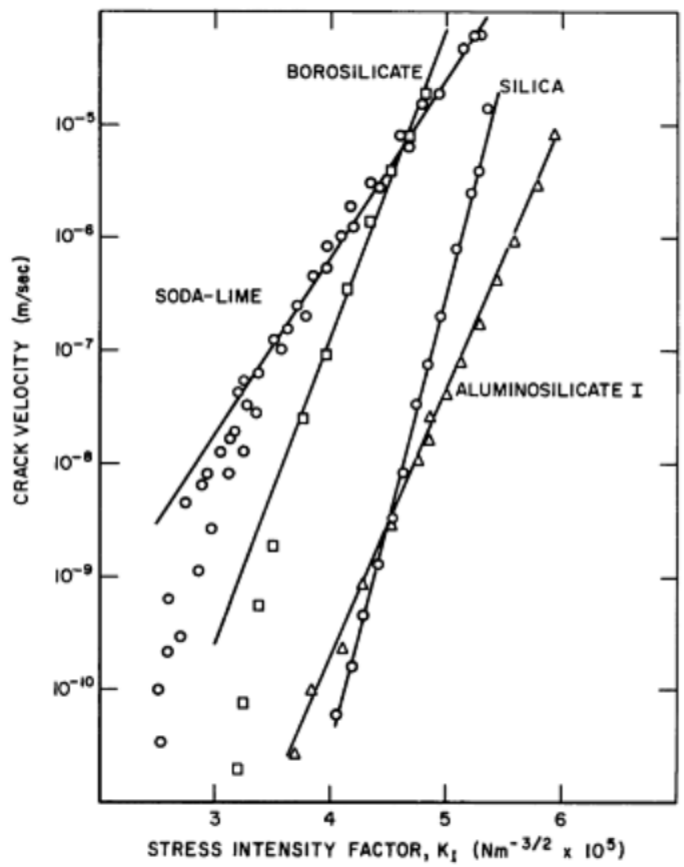


FIG. 20. Effect of glass composition on crack velocity–K_I behavior of various glasses tested in H_2O at room temperature. (After Wiederhorn and Bolz, 1970.)

Velocity of crack vs. stress intensity, K_i curve exponents for fused silica. (Skylab paper reference)

$$K_I := a + b \cdot \ln(v)$$

From skylab paper:

fused silica is designated as C7940 glass

$$a := 6.931 \cdot 10^{-1} \cdot MPa \cdot m^{.5} \qquad b := 1.342 \cdot 10^{-2} \cdot MPa \cdot m^{.5}$$

$$v_{crack}(K_I) := e^{\frac{K_I - a}{b}} \cdot \frac{m}{s}$$

Time to failure:

Use numerical integation script to calculate the duration for a crack of inital size, a_0 to grow to the critical crack length. (.ie. K_i >= K_{IC})

From Stress Corrosion and static fatigue of glass[4]

To perform the numerical integration, the stress intensity factor is calculated for a crack of initial length, L, and a fixed load condition, and the crack velocity, v, is determined from Fig. 2. The time, Δt, for an incremental increase in length, ΔL, is then determined from Δt=ΔL/v, and a new stress intensity factor is calculated from the new crack length, L+ΔL. The process is repeated until K=KIc, and the time to failure is equal to the sum of the time increments, ΣΔt.

$\Delta L := 1 \cdot micron$ crack growth step size per iteration.

$$time_{fail}(\sigma, a_0) := \left| \begin{array}{l} a \leftarrow a_0 \\ t \leftarrow 0 \\ K_i \leftarrow 2 \cdot \sigma \cdot \sqrt{a} \\ \text{while } K_i < K_{IC} \\ \quad \left| \begin{array}{l} t \leftarrow t + \dfrac{\Delta L}{v_{crack}(K_i)} \\ a \leftarrow a + \Delta L \\ K_i \leftarrow 2 \cdot \sigma \cdot \sqrt{a} \end{array} \right. \\ t \end{array} \right.$$

a_0 initial crack size

definition of crack intensity

examples

From:
http://www.lle.rochester.edu/media/publications/lle_review/documents/v61/61_05_Slurry.
The most common polishing agents are Ce02 and Zr02 with mean particle sizes ranging from 0.01 to 3 micron.

The maximu flaw depth can be approximated as three times the particle size.

Using a maximum final polishing compound of 25 microns should produce scratches (cracks) no larger than 75 microns. (This is obvously very conservative.)

$time_{fail}(10 \cdot MPa, 75 \cdot micron) = 2.757 \times 10^4 \cdot yr$

$time_{fail}(10 \cdot MPa, 9 \cdot micron) = 5.312 \times 10^7 \cdot yr$

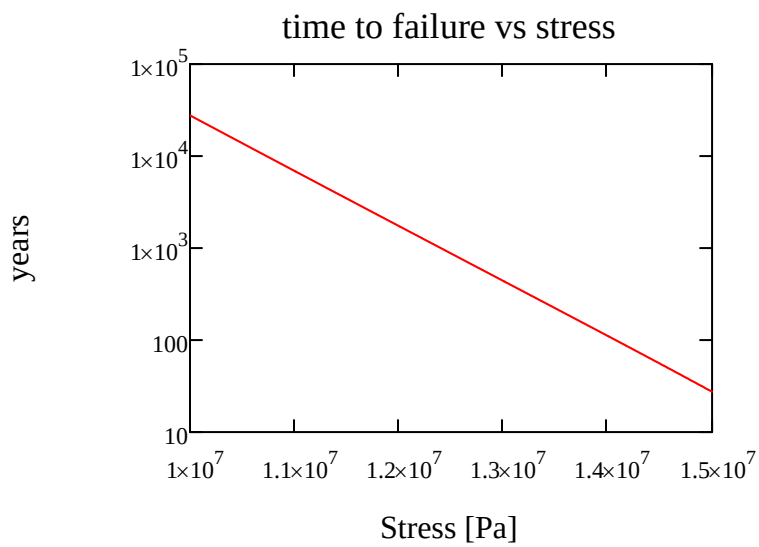
$time_{fail}(15MPa, 75micron) = 27.394 \cdot yr$

$time_{fail}(15MPa, 9micron) = 3.77 \times 10^6 \cdot yr$

$time_{fail}(17MPa, 75micron) = 0.659 \cdot yr$

$time_{fail}(17MPa, 9micron) = 1.371 \times 10^{6} \cdot yr$

Conclusion: Even with 25 micron scratches we can allow 15 MPa for slow crack growth.



Conclusion: Since the crack growth rate for fused silica depends initial crack size care should be taken to remove any scratches in the window deeper than 25 microns. Since this is achievable using common polishing abrasive sizes (0.3 to 9 microns) we just need to keep the stress below 15 MPa to insure a lifetime of >27 years.

Calculate the probability of survival/failure based on measured characteristic strength. This calculation of "fast" rupture is based on test sample statistics using the Weibull constant and characteristic strength.

See:"Characteristic Strength, Weibull modulus and failure probability of fused silica glass" by Claude Klein
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/quartz_weibull.pdf

See: http://www-eng.lbl.gov/~shuman/NEXT/pv_tdr2/quartz-window.pdf

To calculate the effective area under pressure see: Slow Crack Growth and Fracture Toughness of Sapphire for the International Space Station Fluids and Combustion Facility

http://www.ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20060008702_2006006319.pdf

Test results for fused silica.

$\sigma_C := 101MPa$ $\sigma_0 := \sigma_C$ measured characteristic strength of fused silica for 1 square cm samples

$\lambda := 10$ Weibull constant of characteristic strengths

$A_{sample} := 1 \cdot cm^2$ Area of samples

$\upsilon := 0.17$ Fused silica Poisson's ratio

Given example problem by D. Shuman

$R_s := 38 \cdot mm$ Support radius $R_d := 42mm$ window outer radius

ZTF window

$R_d := 9.5 \cdot in$ nominal long radius of window

$D_{offset\_O\_ring} := .68 \cdot in$ O ring offset from edge distance

$R_s := R_d - D_{offset\_O\_ring}$ nominal inside radius of O ring

The maximum stress is only in the center of the window. To correct for this scale the integrated stress over a reduced effective area as follows:

$$A_{effective} := \frac{4 \cdot \pi (1 - \upsilon)}{1 + \lambda} \left(\frac{R_s}{R_d}\right)^2 \frac{2 \cdot R_d^2 (1 + \upsilon) + R_s^2 (1 - \upsilon)}{(3 + \upsilon)(1 + 3 \cdot \upsilon)} = 303.764 \cdot cm^2$$

$$A_{window} := \pi \cdot R_s^2 = 1.577 \times 10^3 \cdot cm^2$$

$$k_2 := \frac{A_{effective}}{A_{window}} = 0.193$$ ratio of window stressed at σ_{max}

In Shuman's example he chooses a 99% survival probability. This means we "allow only 1% of purchased windows to fail a pressure test. This seems reasonable since we will pressure test the windows before inserting the CCD's.

Define probability of survival, Ps

$$\sigma_{max}(P_s) := \sigma_C \cdot \left(\frac{\ln(P_s)}{-k_2 \cdot \frac{A_{window}}{A_{sample}}} \right)^{\frac{1}{\lambda}}$$

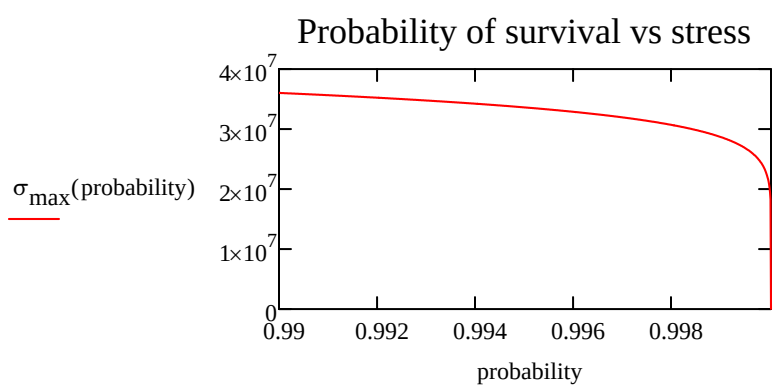
Examples of maximum stress vs. probability of survival.

$\sigma_{max}(.99) = 35.999 \cdot MPa$

$\sigma_{max}(.999) = 28.582 \cdot MPa$

$$P_s(\sigma) := e^{\frac{-A_{effective}}{A_{sample}} \left(\frac{\sigma}{\sigma_C} \right)^{\lambda}}$$

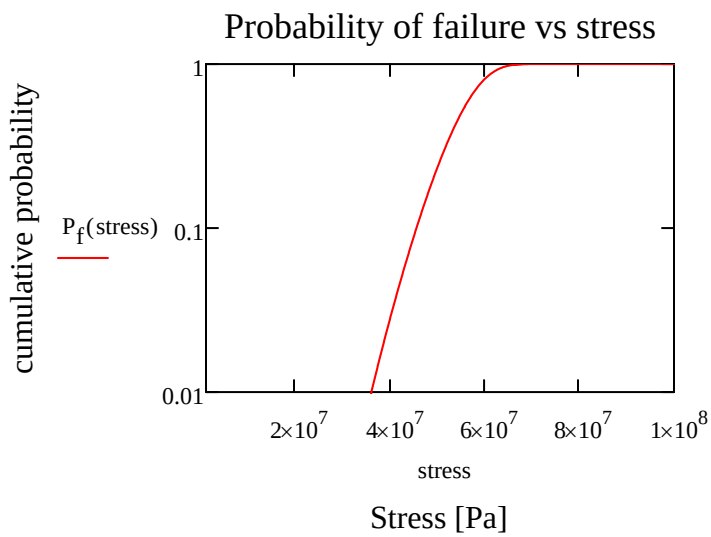
$P_s(20MPa) = 0.9999718411$



Probability of failure, Pf

$P_f(\sigma) := 1 - P_s(\sigma)$

$P_f(10 \cdot MPa) = 2.75 \times 10^{-8}$



Note that this plots scales nicely with that from Klein for our size window!!!

Factor of safety

So for any given probability of failure the factor of safety for any stress can now be calculated:

$$FS(\sigma, P_s) := \frac{\sigma_{max}(P_s)}{\sigma}$$

$FS(10MPa, 0.99) = 3.6$

$FS(8.3MPa, 0.99) = 4.337$ One O-ring

$FS(7.2MPa, 0.99) = 5$ Reduced support under corners.

$FS(9MPa, 0.99) = 4$

$FS(11.2MPa, 0.99) = 3.214$

Conclusion: Moving the O-ring improves the factor of saftey from 4 to 4.8 with a 99% survival probability.

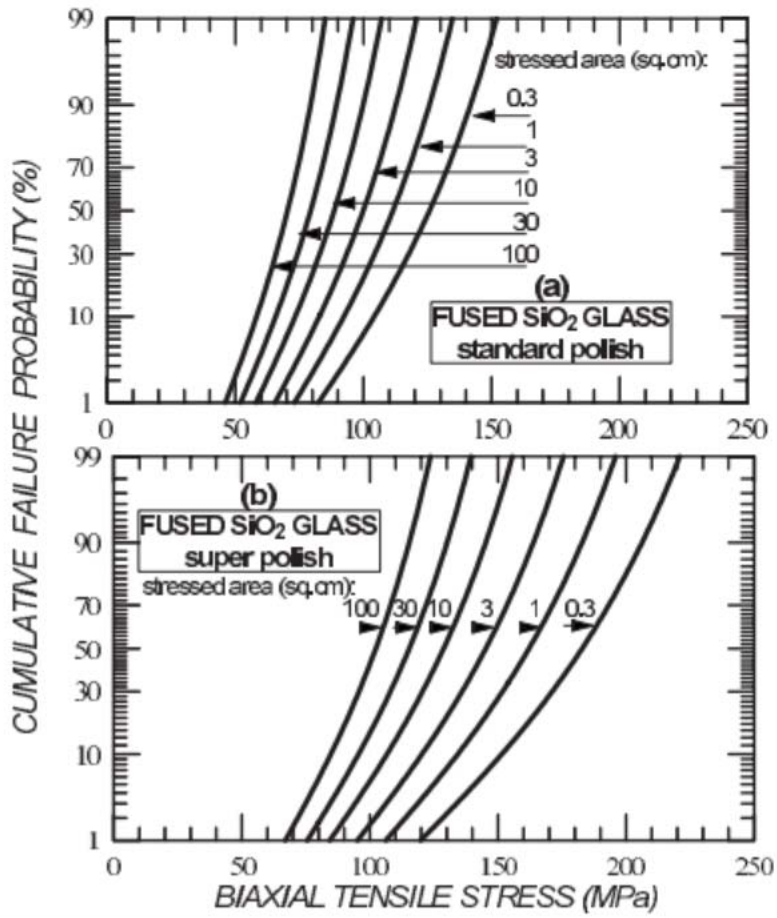


Fig. 6 Cumulative failure probability of fused SiO_2 windows subjected to equibaxial uniform tensile stresses over surface areas ranging from 0.3 to 100 cm^2. The two plots are based on Eq. (11) with m set equal to 10. (a) Standard polish (σ_C=110 MPa) and (b) super polish (σ_C=160 MPa).

$$\operatorname{atan}\left(\frac{.5}{2.5}\right) = 0.197$$

$$315\text{GPa} = 3.15 \times 10^{11}\,\text{Pa}$$

pdf

agrees

agrees

agrees

agrees

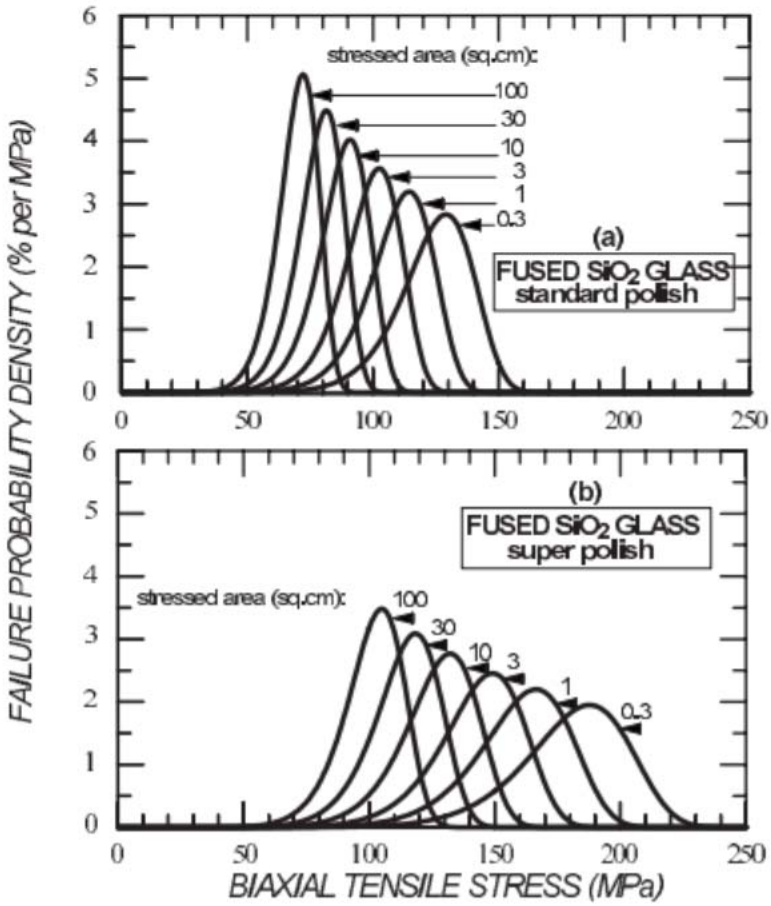


Fig. 7 Failure probability density of fused SiO_2 windows subjected to equibiaxial uniform tensile stresses over surface areas ranging from 0.3 to 100 cm^2. The two plots are based on Eq. (12) with m set equal to 10. (a) Standard polish (σ_C=110 MPa) and (b) super polish (σ_C=160 MPa).